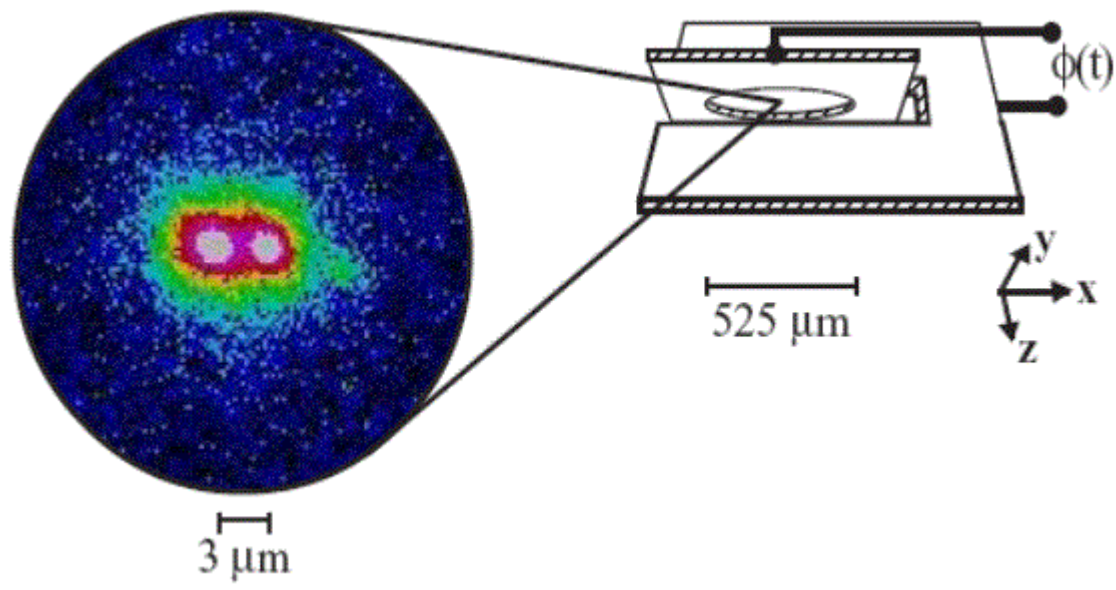
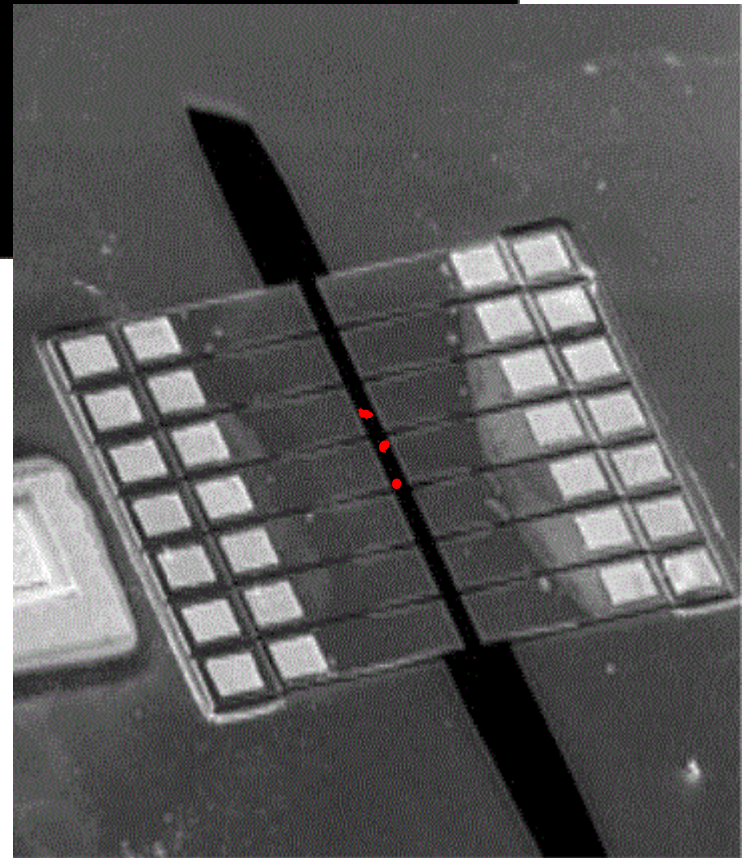
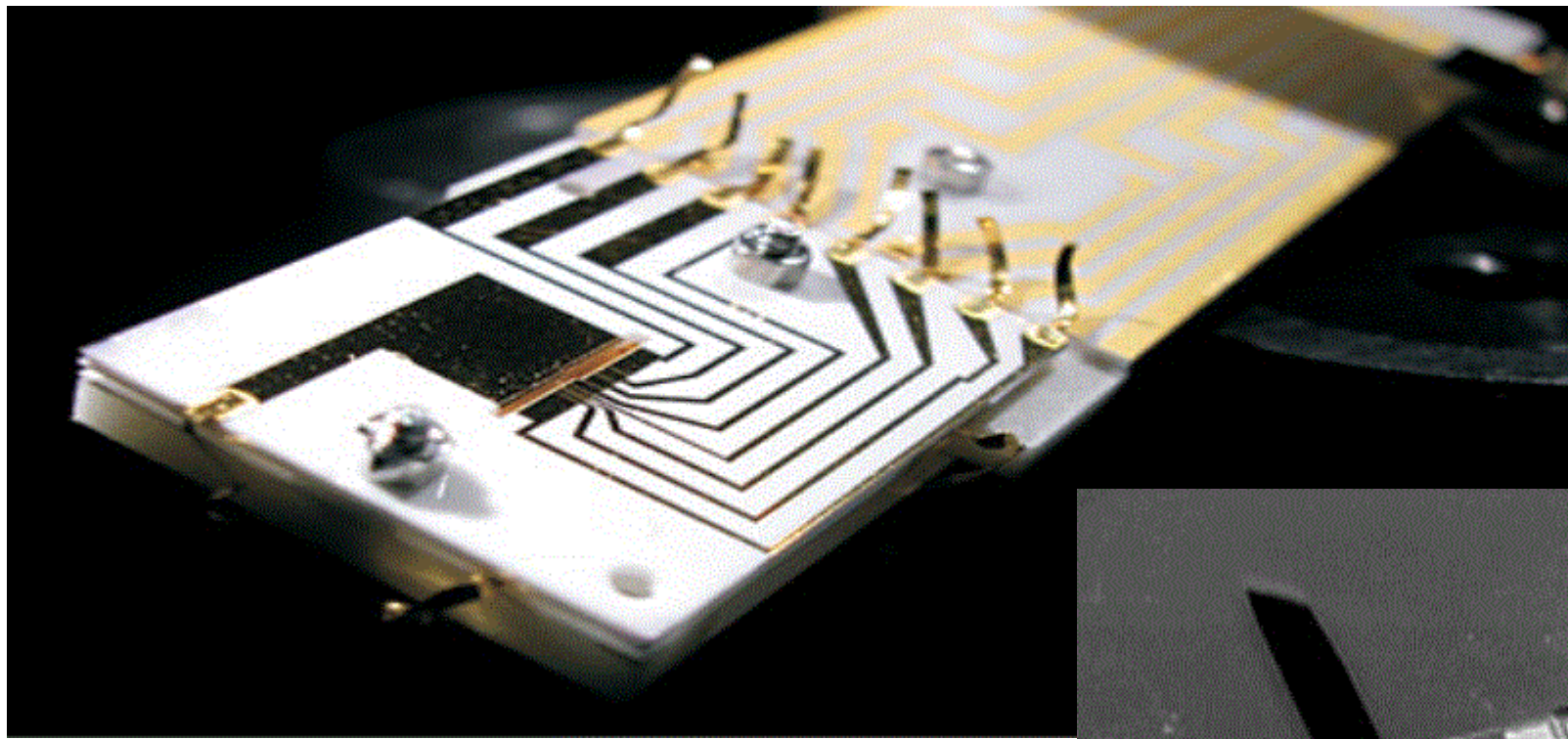
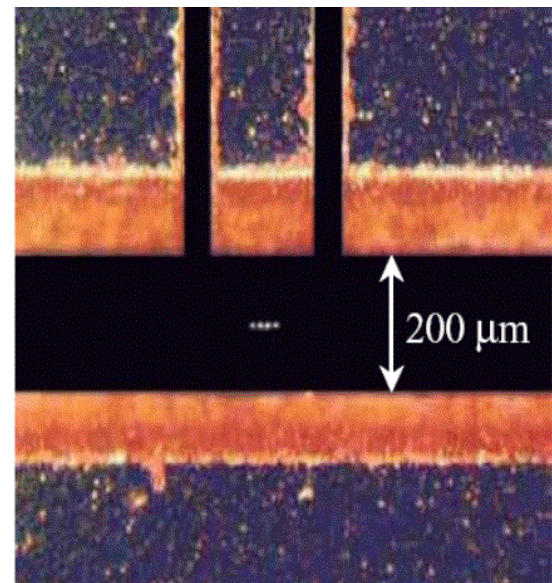
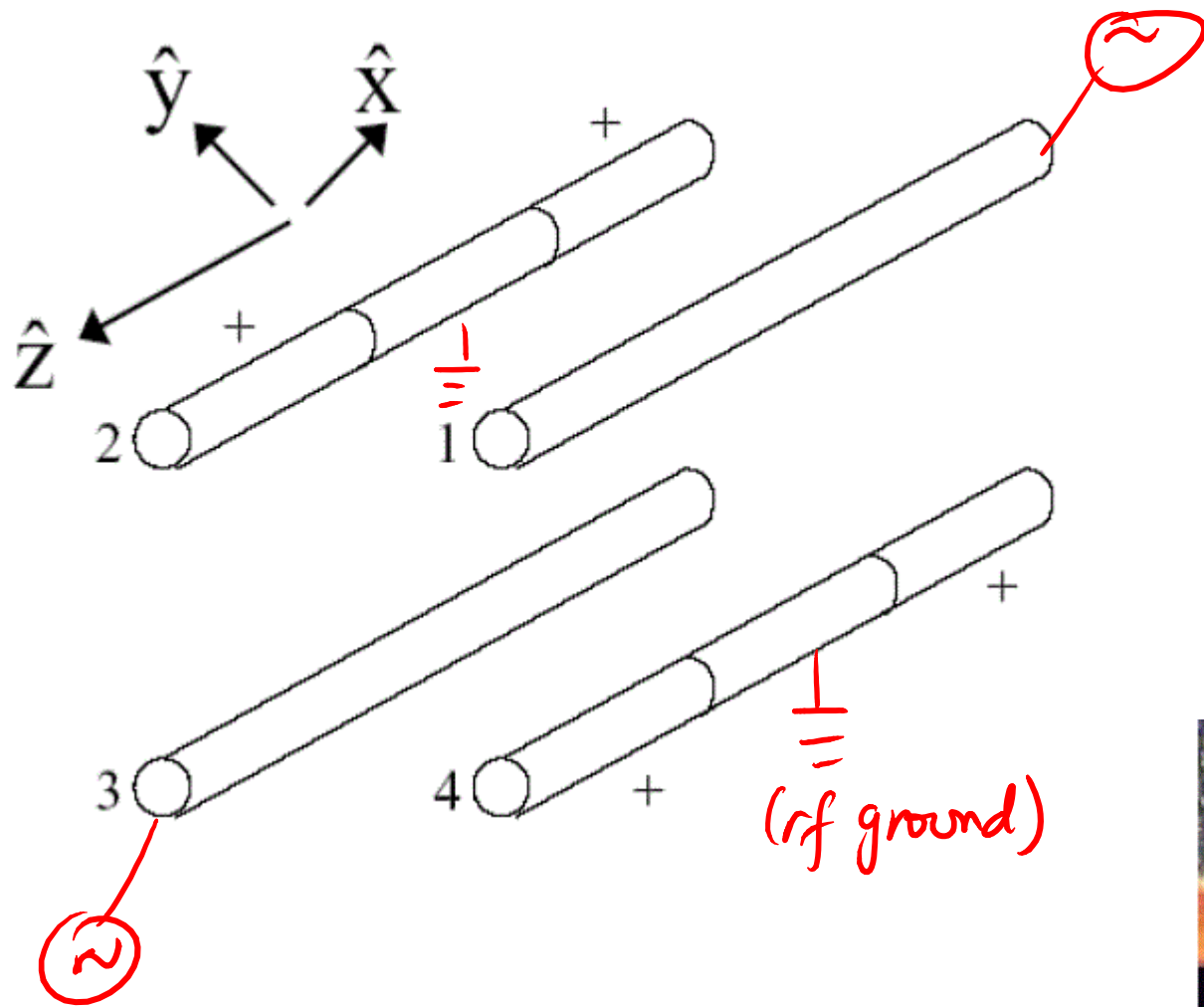




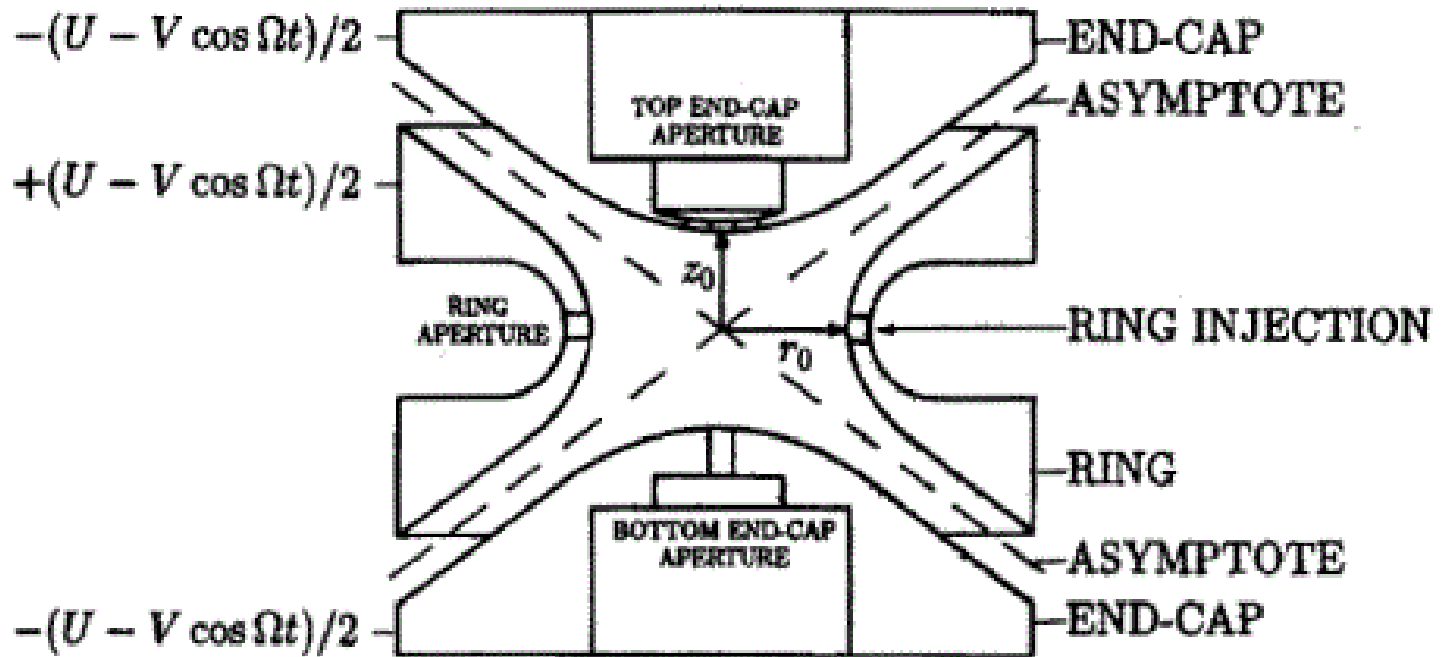


linear Paul trap



<http://www.youtube.com/watch?v=bkYXNeJ8IP0>

<http://www.youtube.com/watch?v=PYpbKSmOnNc>



$$\Phi = \Phi_0 \frac{(x^2 + y^2 - 2z^2)}{2 r_0^2} \cos(\Omega t)$$

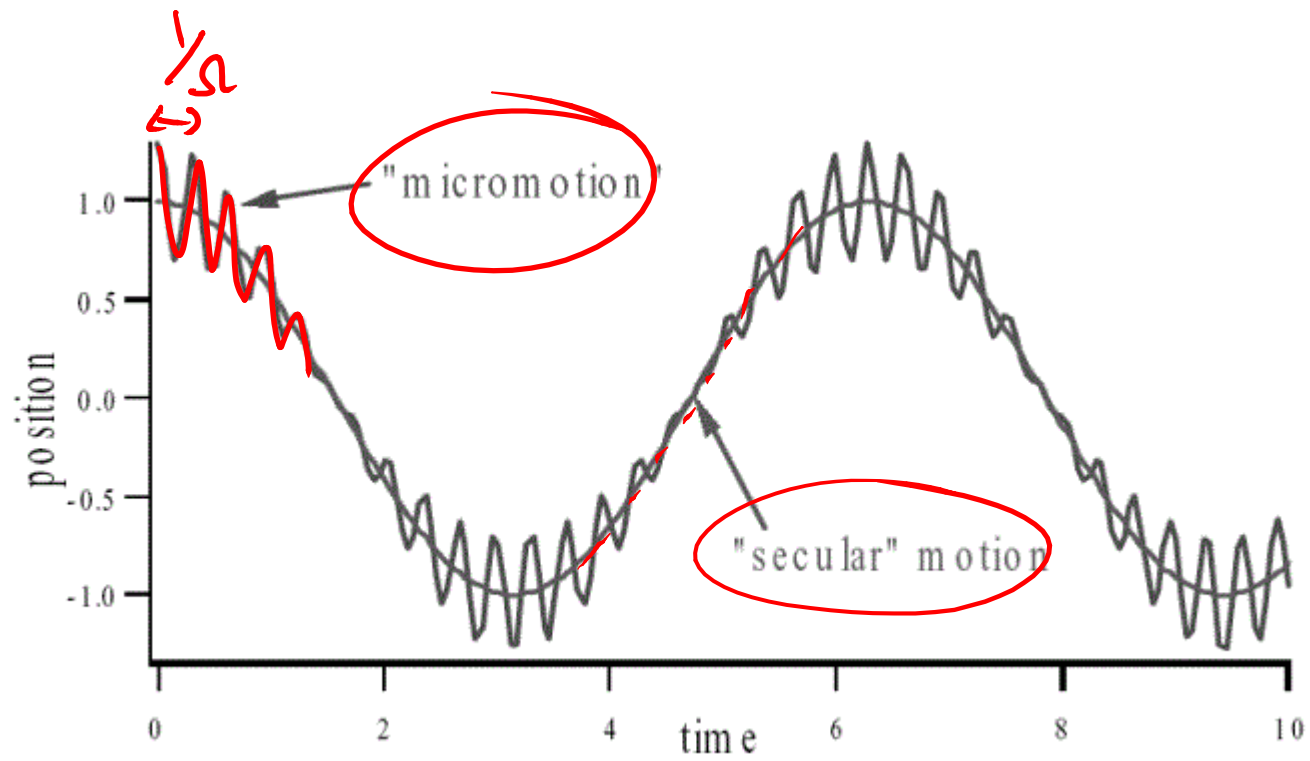
$$\Phi = \Phi_0 \frac{(x^2 + y^2 - 2z^2)}{2 r_0^2} \cos(\Omega t)$$

$$\ddot{r} + \frac{e}{m r_0^2} [\overset{\text{DC}}{U} - \overset{\text{RF}}{V} \cos(\Omega t)] = 0 \qquad \ddot{z} - \frac{2e}{m r_0^2} [U - V \cos(\Omega t)] = 0$$

$$a_z = -2a_r = -\frac{8eU}{m r_0^2 \Omega^2} \qquad q_z = -2q_r = -\frac{4eV}{m z_0^2 \Omega^2} \qquad \zeta = \frac{\Omega t}{2}$$

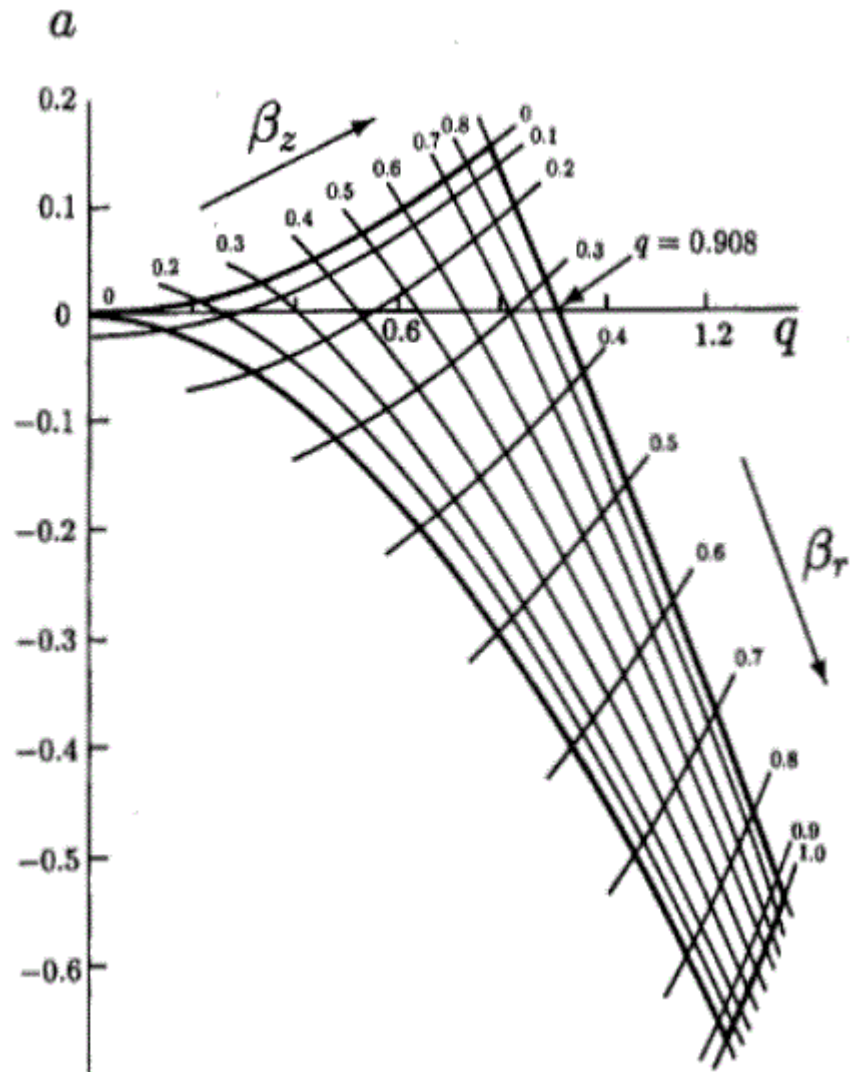
Mathieu equations

$$\frac{\partial^2 r}{\partial \zeta^2} + [a_r - 2q_r \cos(2\zeta)]r = 0 \qquad \frac{\partial^2 z}{\partial \zeta^2} + [a_z - 2q_z \cos(2\zeta)]z = 0$$



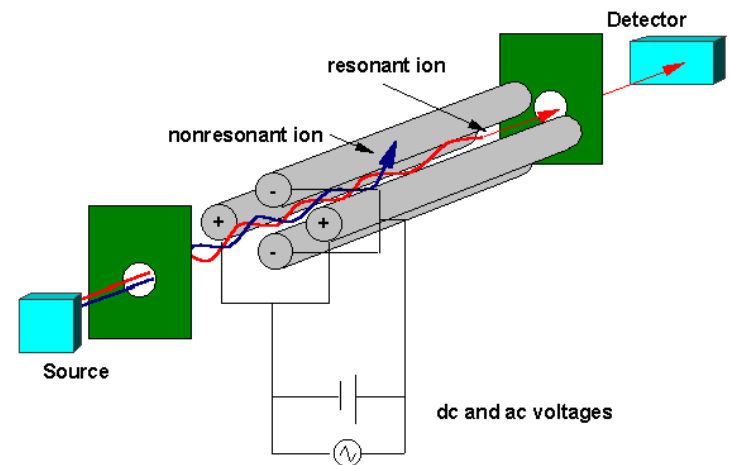
$$a_z = -2a_r = -\frac{8eU}{mr_0^2\Omega^2}$$

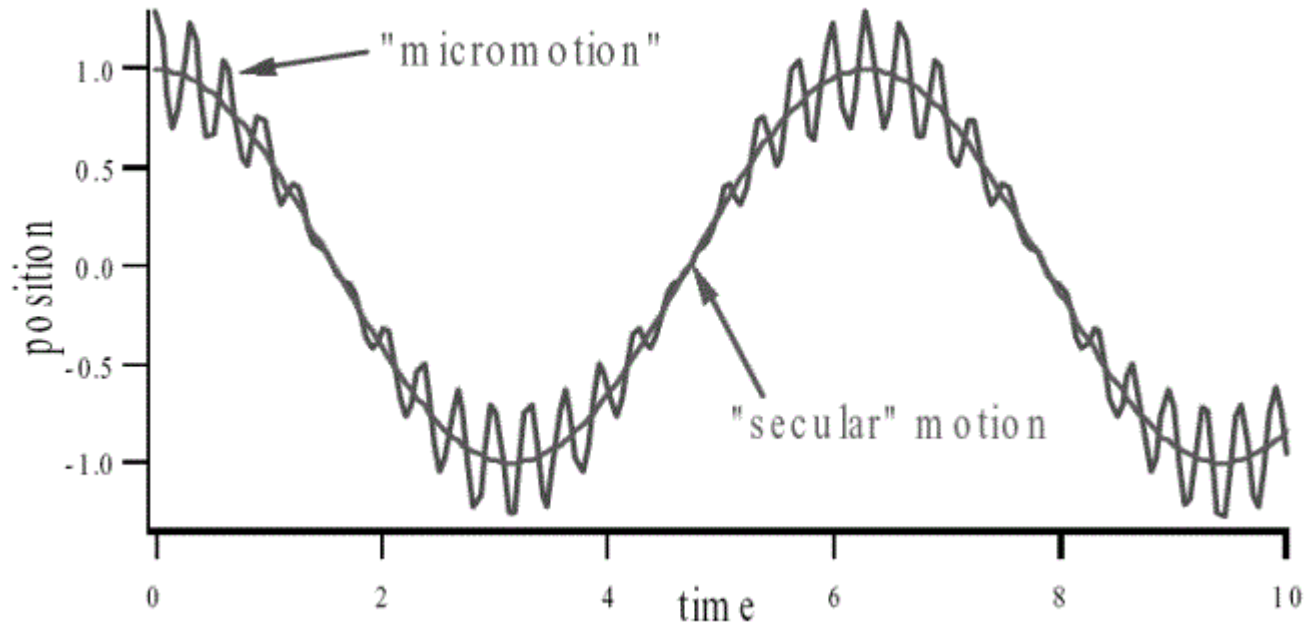
$$q_z = -2q_r = -\frac{4eV}{mr_0^2\Omega^2}$$



$$\beta_z = \sqrt{a_z^2 + \frac{q_z^2}{2}}$$

$$\beta_r = \sqrt{a_r^2 + \frac{q_r^2}{2}}$$



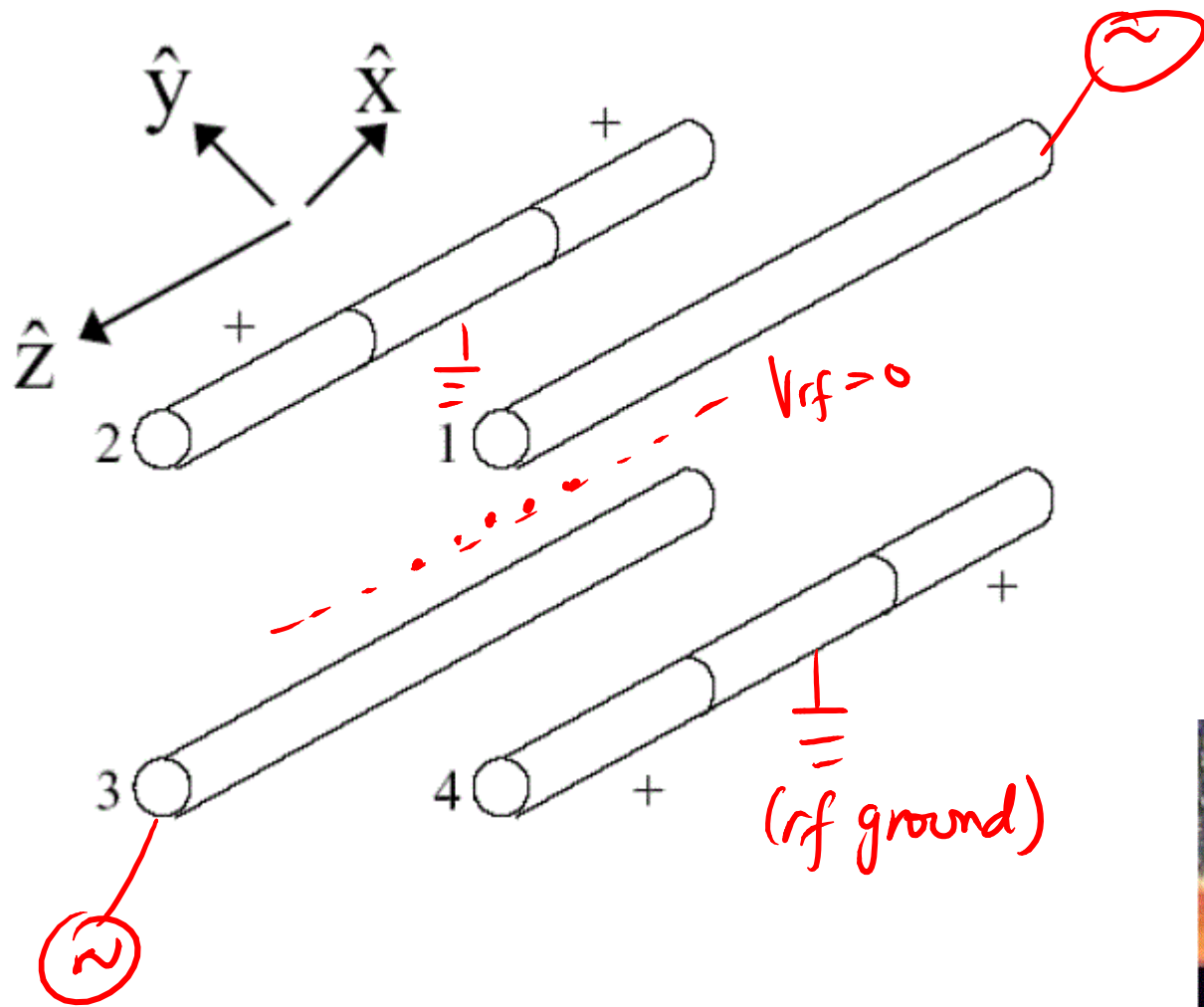


$$z = Z + \delta \quad \dot{\delta} \gg \dot{z}$$

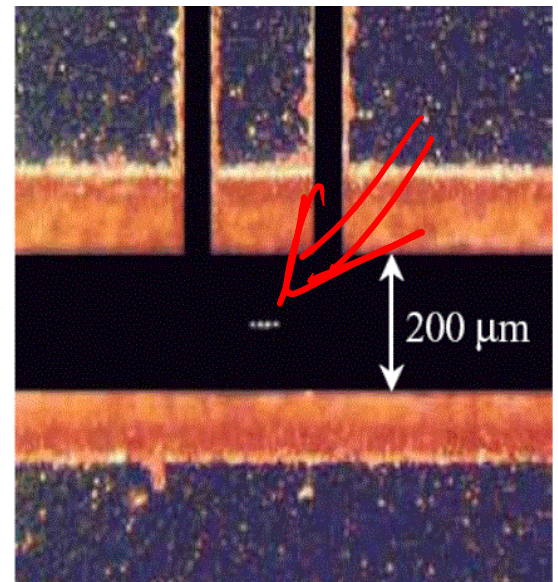
$$\delta \approx -\frac{q_z Z}{2} \cos(2\zeta) \quad \omega \sim 2\pi \text{ (few MHz)}$$

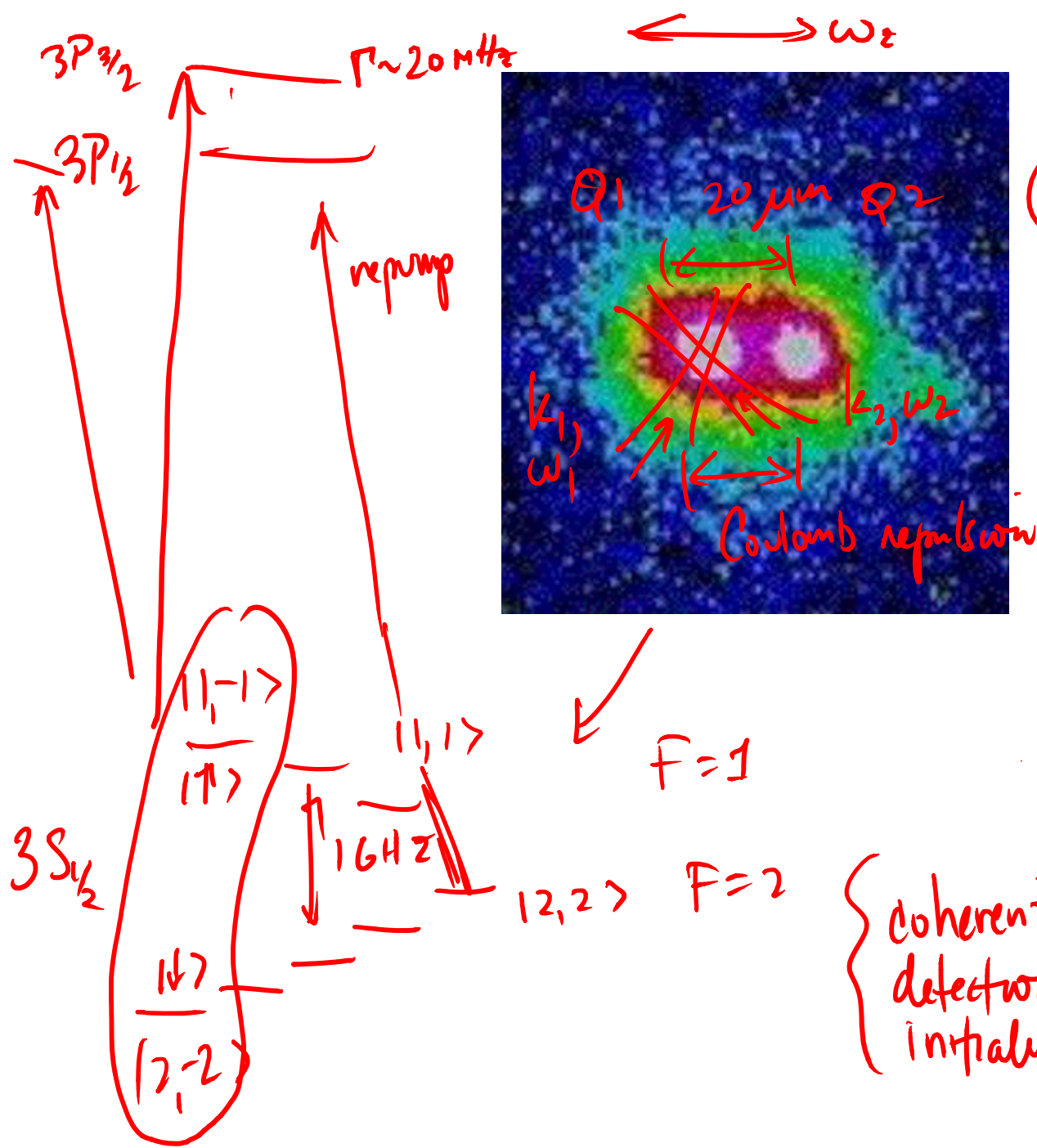
$$\left\langle \frac{d^2 Z}{dt^2} \right\rangle_{\Omega} \approx -\frac{\beta_z^2 \Omega^2}{4} Z$$

~~ω^2~~

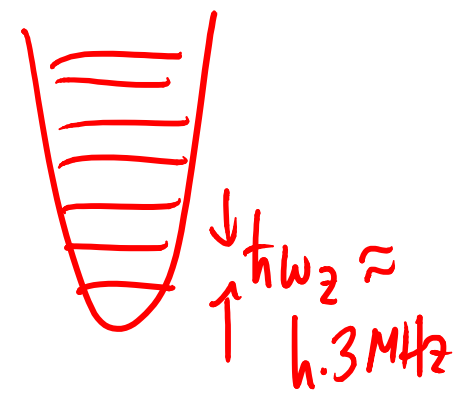


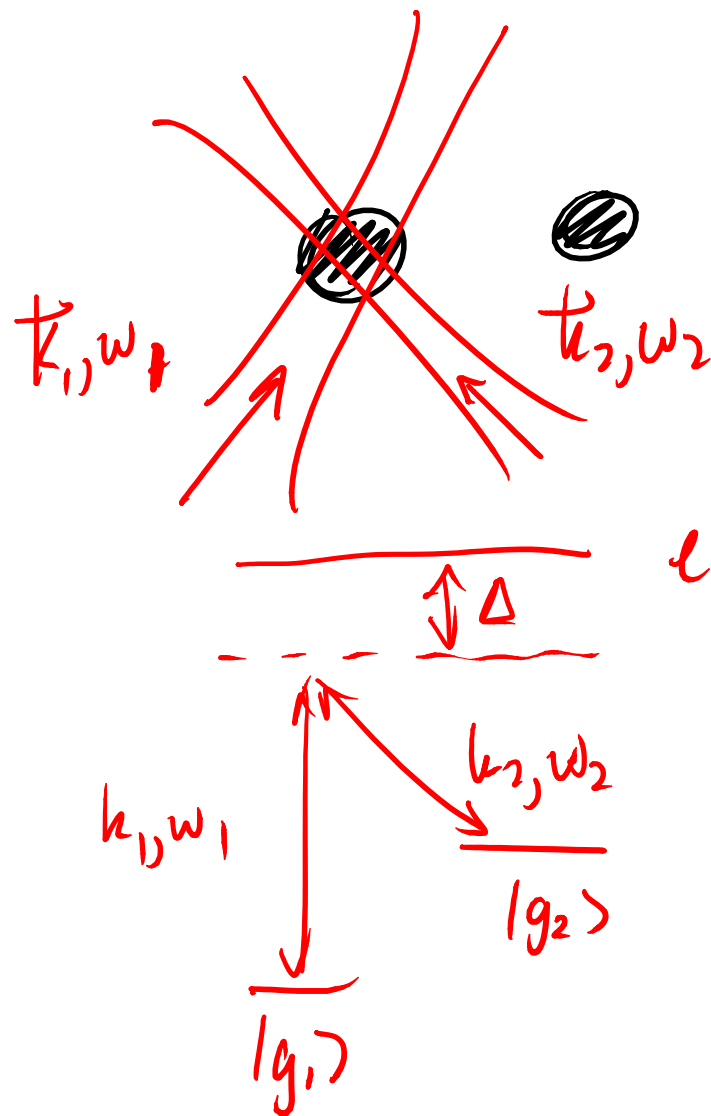
linear Paul trap





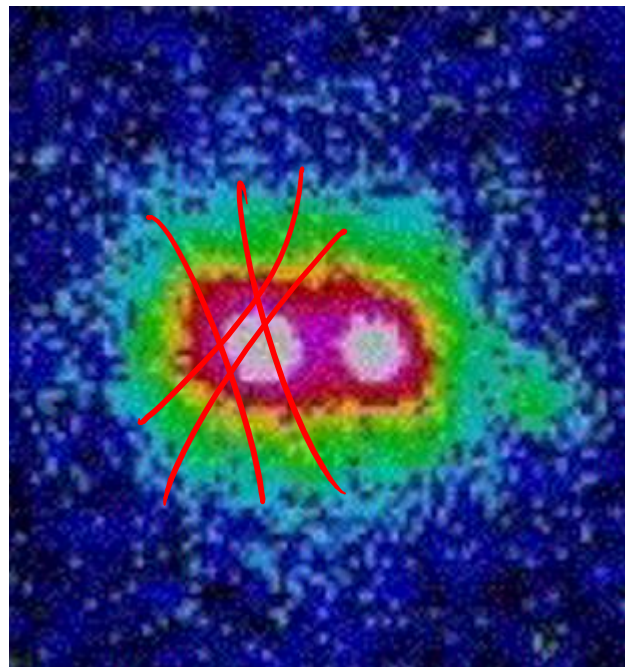
- ① Single qubit rotations
 - ② 2 qubit gate
- 9Be^+

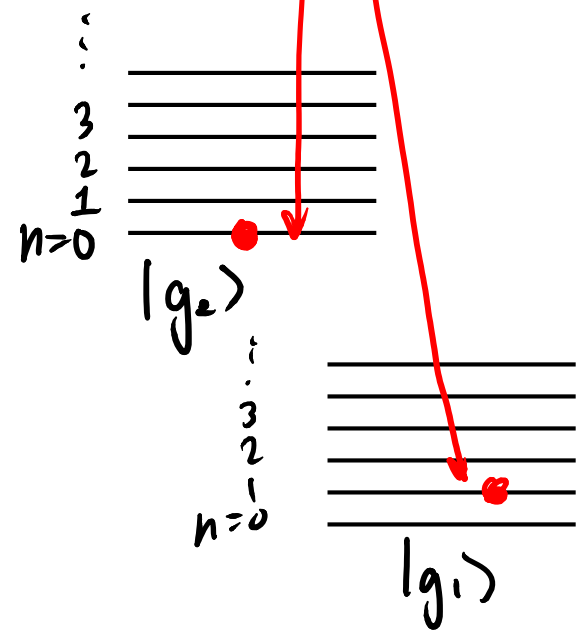




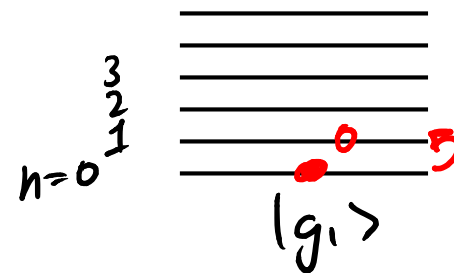
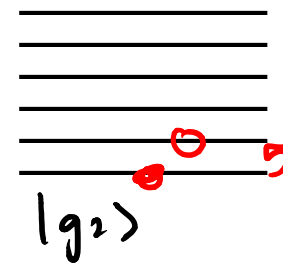
stimulated Raman
transition

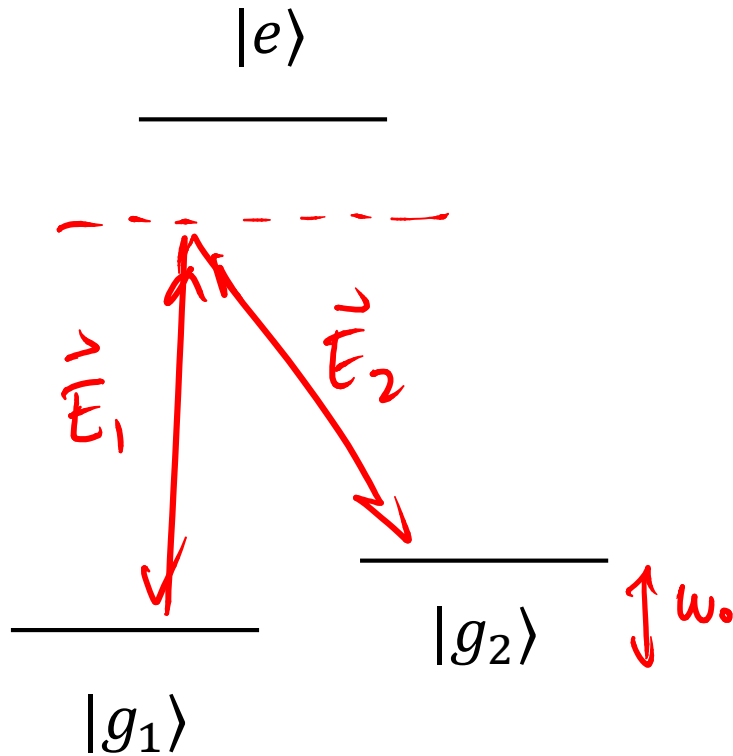
$|e\rangle$ _____





$\Delta n = 1$





$$\vec{E}_1 = E_1 e^{i k_1 z} e^{-i \omega_1 t} \hat{x}$$

$$\vec{E}_2 = E_2 e^{i k_2 z} e^{-i \omega_2 t} \hat{x}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = e^{i k z} E (e^{i \omega_1 t} + e^{i \omega_2 t}) \hat{x}$$

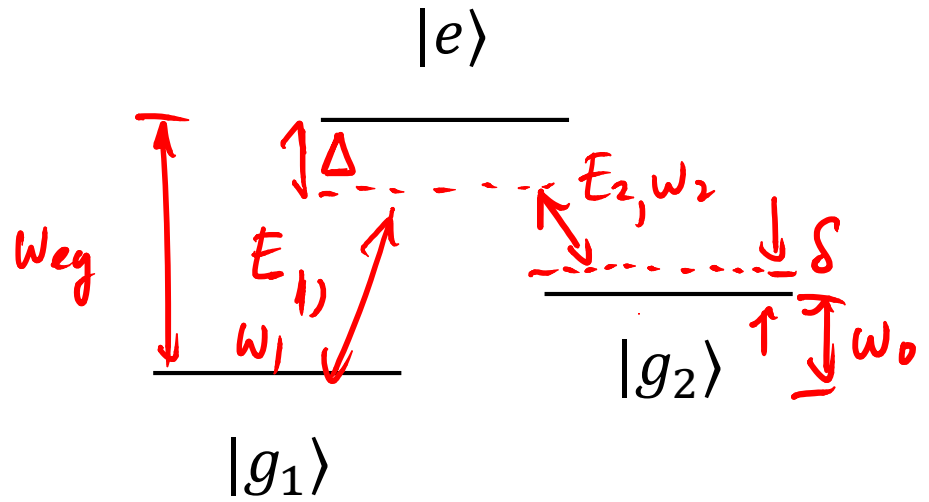
$$|\vec{E}|^2 = 2E^2 + 2E^2 \cos[(\omega_1 - \omega_2)t]$$

$$\omega_1 - \omega_2 \approx \omega_0$$

$$e^{ikx} \approx 1$$

$$\vec{E}_1 = E_1(e^{-i\omega_1 t} + e^{i\omega_1 t})\hat{e}_1$$

$$\vec{E}_2 = E_1(e^{-i\omega_2 t} + e^{i\omega_2 t})\hat{e}_2$$



$$H_I' = -\frac{e}{2}\{\langle e|\hat{e}_1 \cdot \vec{r}|g_1\rangle |e\rangle\langle g_1|e^{i\omega_{eg}t}e^{-i\omega_1t}E_1$$

$$+\{\langle e|\hat{e}_2 \cdot \vec{r}|g_2\rangle |e\rangle\langle g_2|e^{i\omega_{eg}t}e^{-i\omega_2t}e^{-i\omega_0t}E_2\} + hc$$

$$\Delta = \omega_{eg} - \omega_1$$

$$\Delta \gg \delta$$

$$\Delta + \delta = \omega_{eg} - \omega_2 - \omega_0$$

$$|\psi\rangle = c_1|g_1\rangle + c_2|g_2\rangle + c_e|e\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = H_I' |\psi\rangle$$

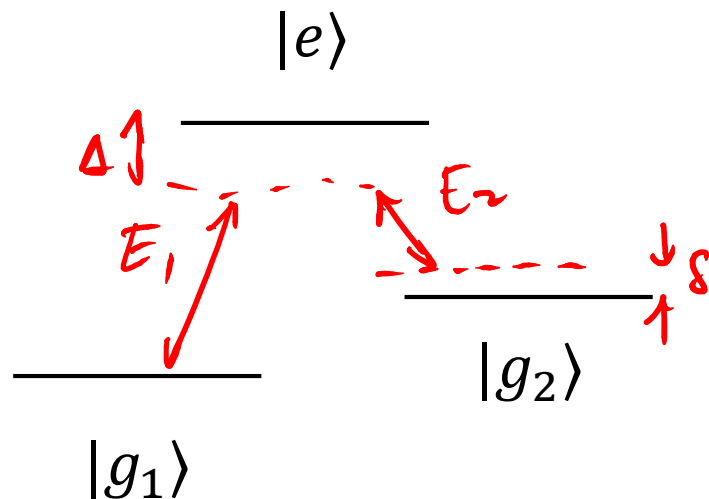
RWA



$$\dot{c}_1 = i \frac{eE_1^*}{2\hbar} e^{-i\Delta t} \langle g_1 | \hat{\epsilon}_1 \cdot \vec{r} | e \rangle c_e$$

$$\dot{c}_2 = i \frac{eE_2^*}{2\hbar} e^{-i(\Delta+\delta)t} \langle g_1 | \hat{\epsilon}_2 \cdot \vec{r} | e \rangle c_e$$

$$\dot{c}_e = i \frac{e}{2\hbar} \left[e^{i\Delta t} \langle e | \hat{\epsilon}_1 \cdot \vec{r} | g_1 \rangle E_1 c_1 + e^{i(\Delta+\delta)t} \langle e | \hat{\epsilon}_2 \cdot \vec{r} | g_1 \rangle E_2 c_2 \right]$$

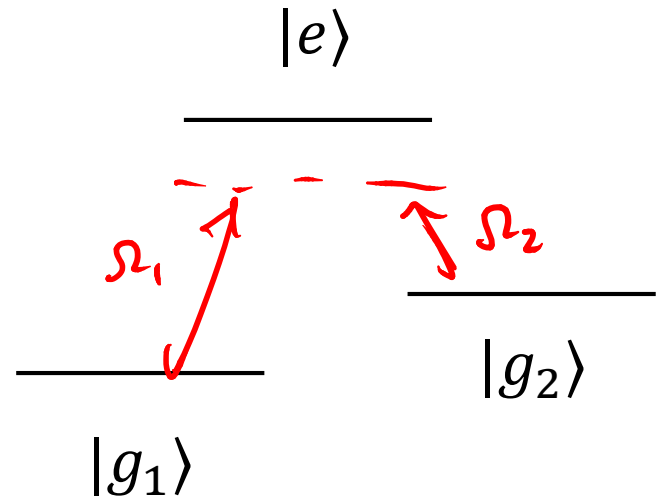


$$\dot{c}_1 = i \frac{eE_1^*}{2\hbar} e^{-i\Delta t} \langle g_1 | \hat{\epsilon}_1 \cdot \vec{r} | e \rangle c_e$$

Ω_1

$$\dot{c}_2 = i \frac{eE_2^*}{2\hbar} e^{-i(\Delta+\delta)t} \langle g_1 | \hat{\epsilon}_2 \cdot \vec{r} | e \rangle c_e$$

Ω_2



$$\dot{c}_e = i \frac{e}{2\hbar} \left[e^{i\Delta t} \langle e | \hat{\epsilon}_1 \cdot \vec{r} | g_1 \rangle E_1 c_1 + e^{i(\Delta+\delta)t} \langle e | \hat{\epsilon}_2 \cdot \vec{r} | g_2 \rangle E_2 c_2 \right]$$

Ω_1 Ω_2

$$\Omega_1 = \frac{eE_1}{2\hbar} \langle e | \hat{\epsilon}_1 \cdot \vec{r} | g_1 \rangle$$

$$\Omega_2 = \frac{eE_2}{2\hbar} \langle e | \hat{\epsilon}_2 \cdot \vec{r} | g_2 \rangle$$

$$c'_e = e^{-i\Delta t} c_e$$

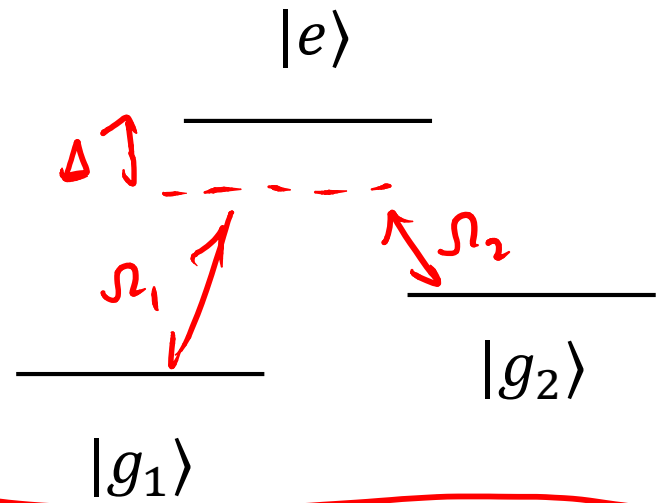
$$c'_1 = c_1$$

$$c'_2 = c_2$$

$$\dot{c}_e = e^{i\Delta t} \dot{c}'_e + i\Delta e^{i\Delta t} c'_e$$

$$\dot{c}'_1 = i \frac{eE_1^*}{2\hbar} \langle g_1 | \hat{\epsilon}_1 \cdot \vec{r} | e \rangle c'_e$$

$$\dot{c}'_2 = i \frac{eE_2^*}{2\hbar} e^{-i\delta t} \langle g_1 | \hat{\epsilon}_2 \cdot \vec{r} | e \rangle c'_e$$



adiabatic elimination

~~$$\dot{c}'_e + i\Delta c'_e = i \frac{e}{2\hbar} [\langle e | \hat{\epsilon}_1 \cdot \vec{r} | g_1 \rangle E_1 c'_1 + e^{i\delta t} \langle e | \hat{\epsilon}_2 \cdot \vec{r} | g_2 \rangle E_2 c'_2]$$~~

$$\Delta \gg \Omega_1, \Omega_2, \delta$$

$$c'_e = \frac{1}{\Delta} (\Omega_1 c_1 + \Omega_2 e^{-i\delta t} c_2)$$

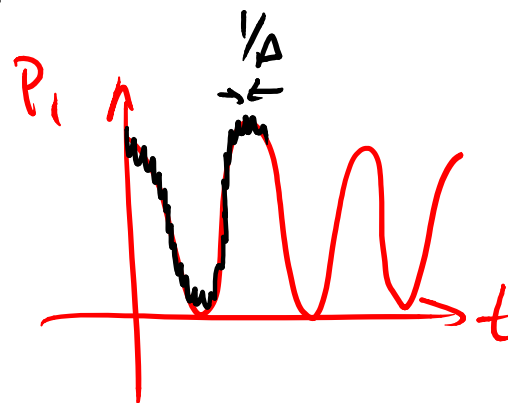
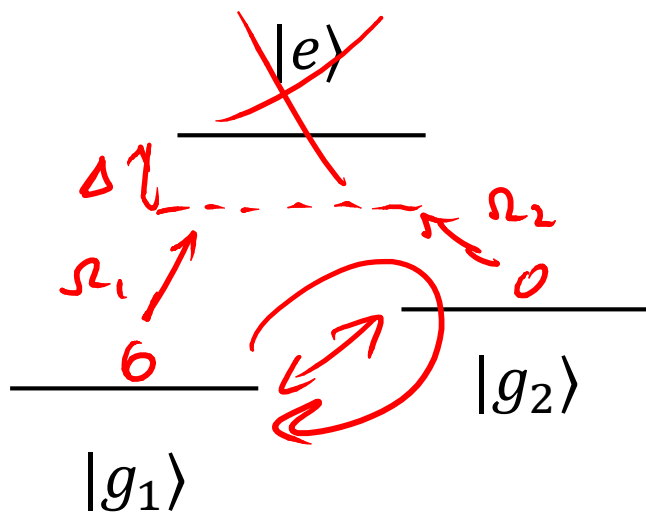
$$\dot{c}_1 = \frac{i|\Omega_1|^2}{\Delta} c_1 + i \frac{\Omega_1^* \Omega_2}{\Delta} c_2 e^{i\delta t}$$

$$\dot{c}_2 = \frac{i|\Omega_2|^2}{\Delta} c_2 + i \frac{\Omega_1^* \Omega_2}{\Delta} c_1 e^{-i\delta t}$$

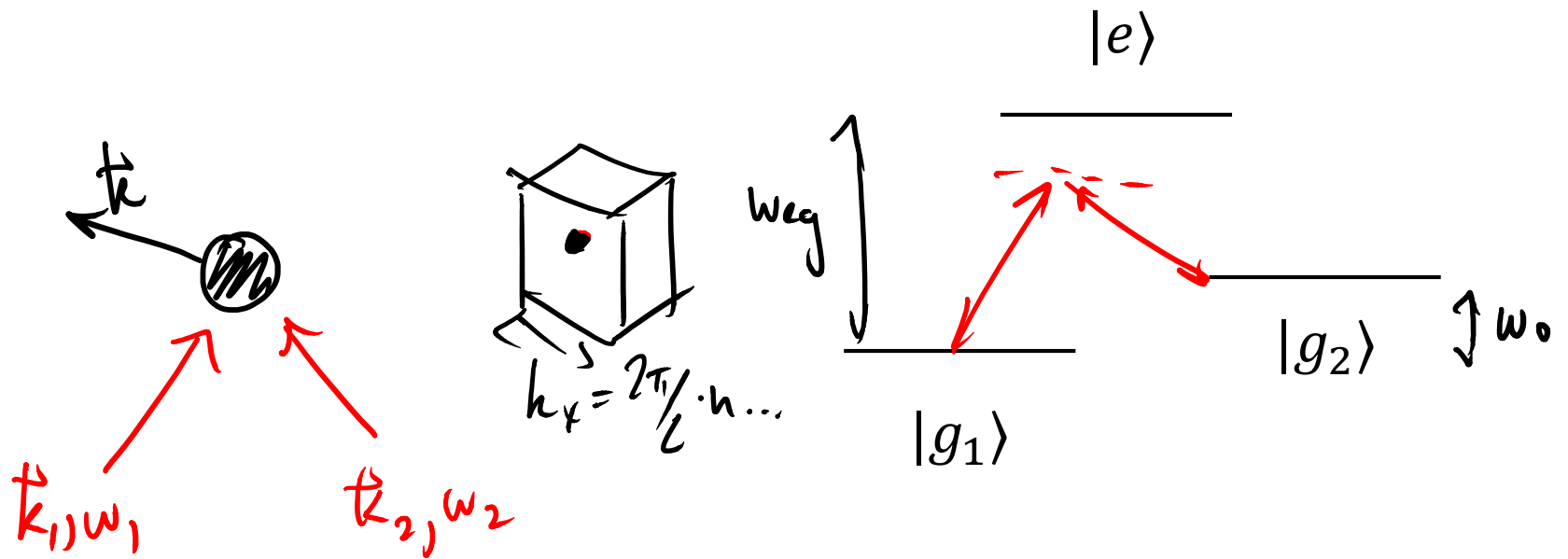
$$\dot{c}_1 = \frac{i|\Omega_1|^2}{\Delta} c_1 + i \frac{\Omega_1^* \Omega_2}{\Delta} c_2 e^{i\delta t} \quad \dot{c}_2 = \frac{i|\Omega_2|^2}{\Delta} c_2 + i \frac{\Omega_1^* \Omega_2}{\Delta} c_1 e^{-i\delta t}$$

AC Stark shift

AC Stark shift



$$\Omega = \frac{\Omega_1 \Omega_2^*}{\Delta}$$



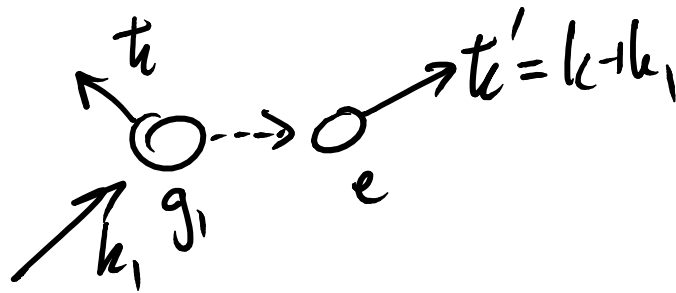
$$H_0 = \frac{\hbar^2 k^2}{2m} + \hbar\omega_0 |g_2\rangle\langle g_2| + \hbar\omega_{eg} |e\rangle\langle e|$$

$$|\psi\rangle = \sum_{\vec{k}} \left\{ c_{1\vec{k}} |g_1\rangle + c_{2\vec{k}} |g_2\rangle + c_{e\vec{k}} |e\rangle \right\} |\vec{k}\rangle$$

$$\dot{c}_{1\vec{k}} = i \frac{eE_1^*}{2\hbar} e^{-i\Delta t} \sum_{\vec{k}} e^{\frac{i(E_k - E'_k)}{\hbar} t} \langle g_1 \vec{k} | \hat{\epsilon}_1 \cdot \vec{r} e^{-i\vec{k}_1 \cdot \vec{r}} | e \vec{k}' \rangle c_{e\vec{k}'}$$

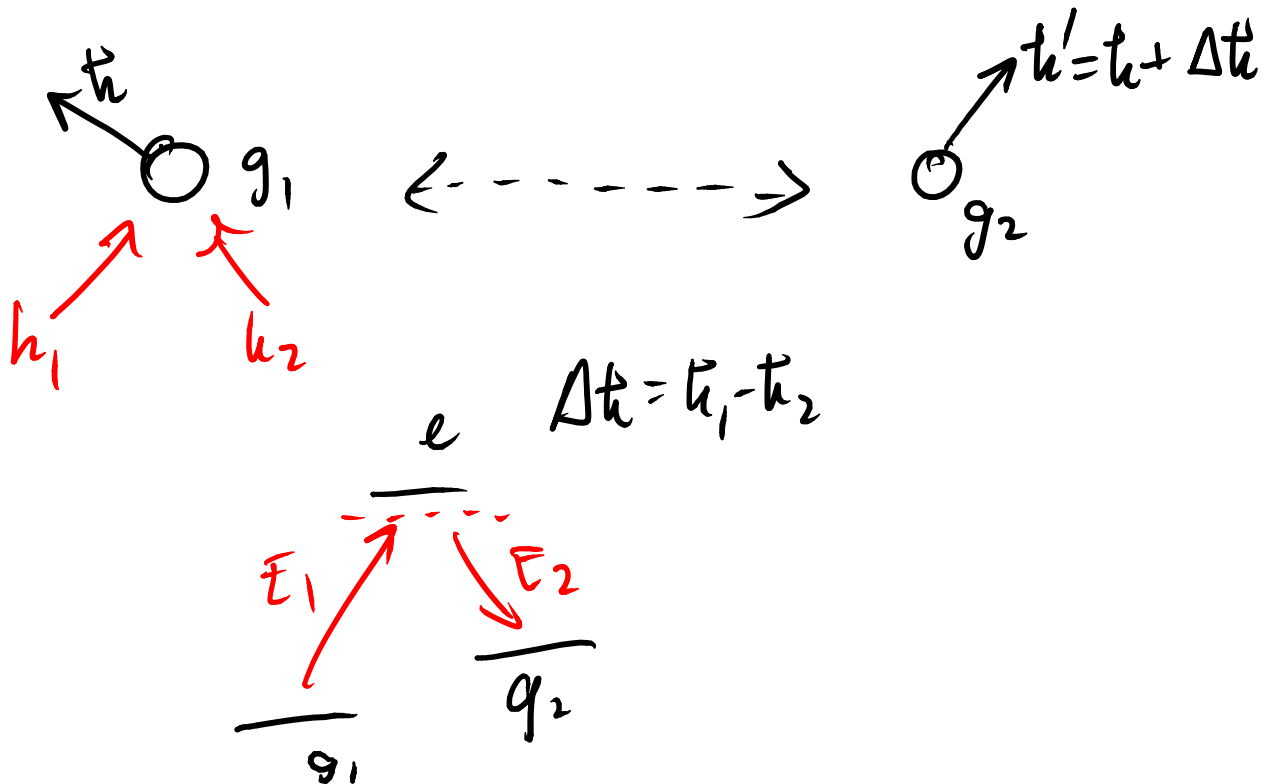
$$\frac{\langle g_1 | \hat{\epsilon}_1 \cdot \vec{r} | e \rangle \langle \vec{k} | e^{-i\vec{k}_1 \cdot \vec{r}} | \vec{k}' \rangle}{\delta_{\vec{k}', \vec{k} + \vec{k}_1}}$$

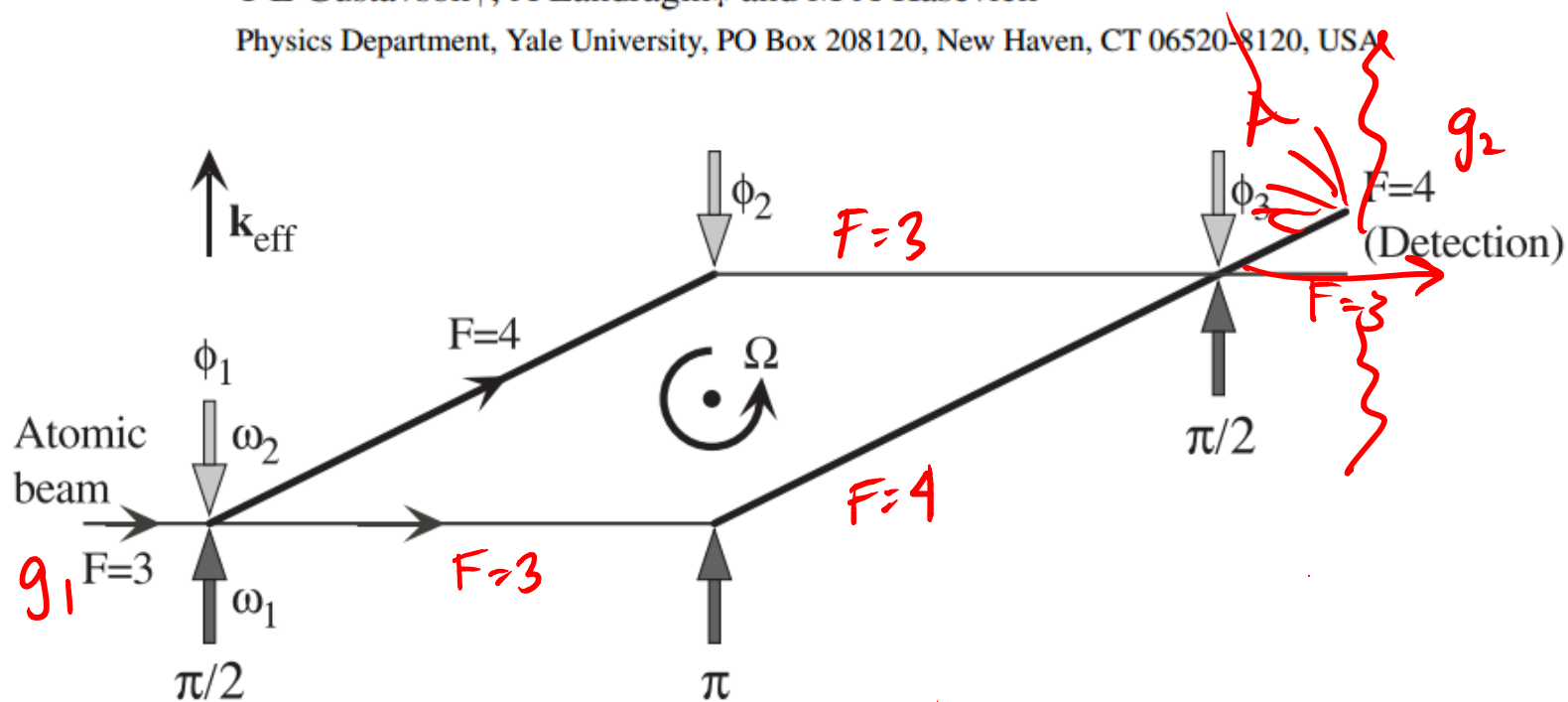
$$\dot{c}_{1\vec{k}} = i \frac{eE_1^*}{2\hbar} e^{-i\Delta t} e^{-\frac{i\hbar^2 k_1^2}{2m}} \langle g_1 | \hat{\epsilon}_1 \cdot \vec{r} | e \rangle c_{e, \vec{k} + \vec{k}_1}$$

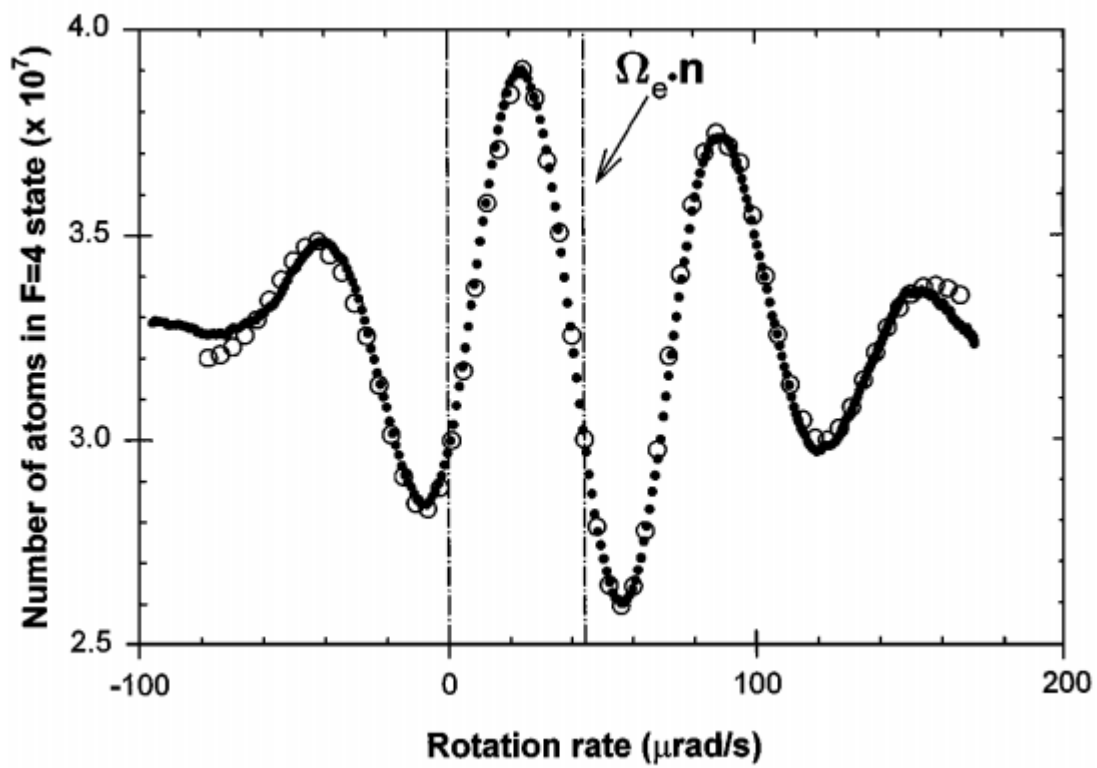


$$\dot{c}_{1,\vec{k}} = \frac{i|\Omega_1|^2}{\Delta} c_{1,\vec{k}} + i \frac{\Omega_1^* \Omega_2}{\Delta} e^{i\delta t} e^{\frac{i\hbar}{2m}(k_2^2 - k_1^2)t} c_{2,\vec{k}+\vec{k}_1-\vec{k}_2}$$

$$\dot{c}_{2,\vec{k}+\vec{k}_1-\vec{k}_2} = \frac{i|\Omega_2|^2}{\Delta} c_{2,\vec{k}+\vec{k}_1-\vec{k}_2} + i \frac{\Omega_1 \Omega_2^*}{\Delta} e^{i\delta t} e^{-\frac{i\hbar}{2m}(k_2^2 - k_1^2)t} c_{1,\vec{k}}$$

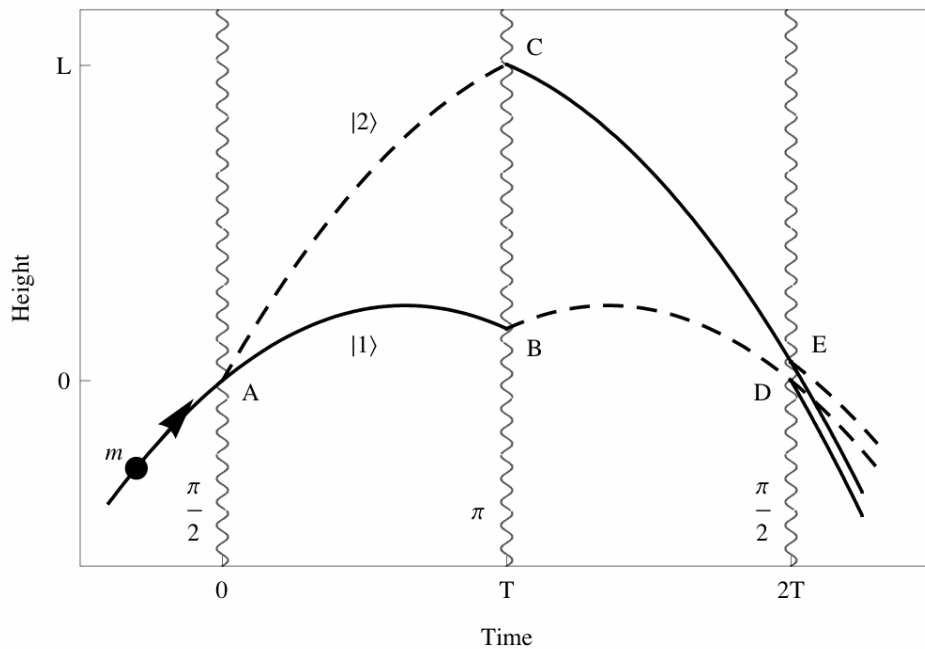






$$\Omega \sim 8 \times 10^{-6} \Omega_E$$

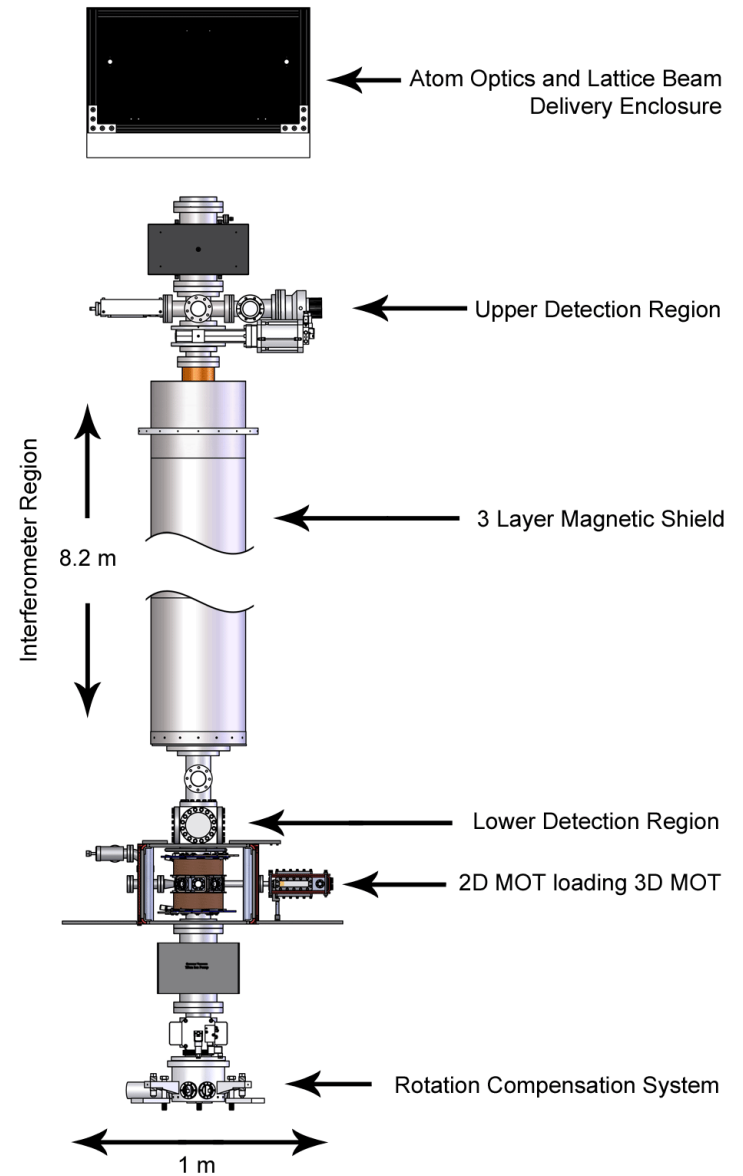




These lead to a sensitivity to the differential acceleration between ^{87}Rb and ^{85}Rb of

$$(g_{87} - g_{85})/g \sim \delta\varphi/\Delta\varphi \sim 10^{-15}$$

To use a physical comparison, this corresponds to detecting the gravitational attraction of a can of soda about 1 meter away.

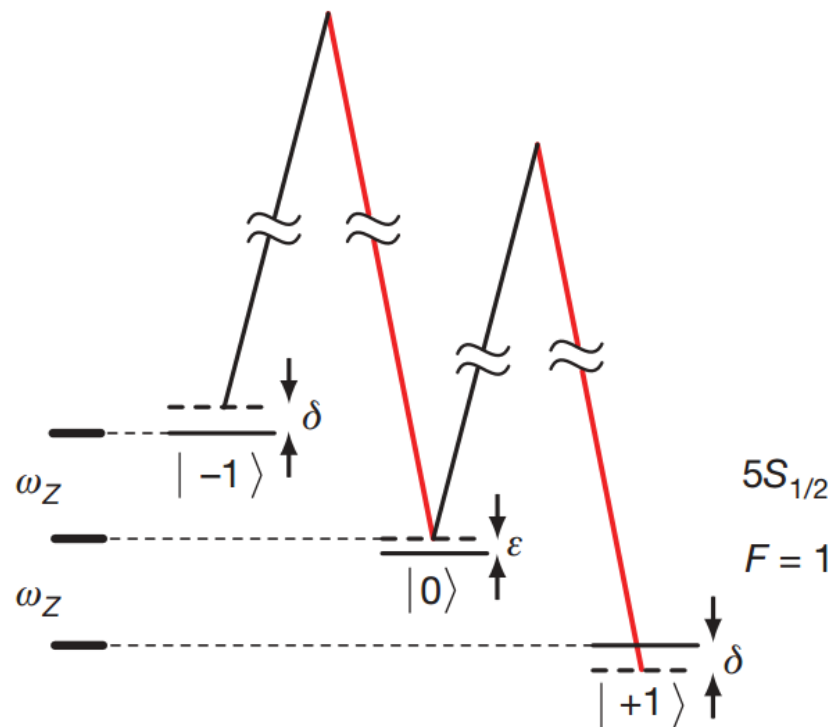
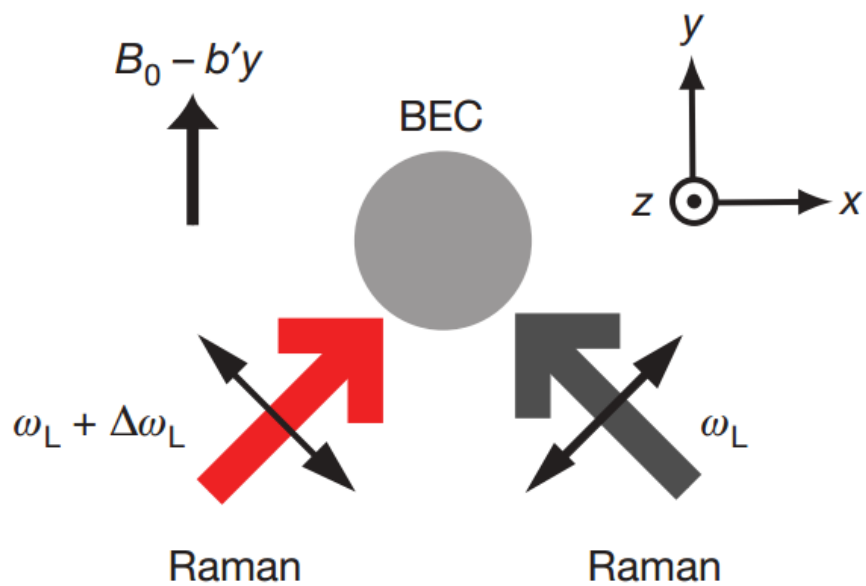


LETTERS

Synthetic magnetic fields for ultracold neutral atoms

Y.-J. Lin¹, R. L. Compton¹, K. Jiménez-García^{1,2}, J. V. Porto¹ & I. B. Spielman¹

$$H = \frac{\hbar^2}{2m} \left(\vec{k} - \frac{q\vec{A}}{\hbar} \right)^2$$



$$\dot{c}_{1,\vec{k}} = \frac{i|\Omega_1|^2}{\Delta} c_{1,\vec{k}} + i \frac{\Omega_1^* \Omega_2}{\Delta} e^{i\delta t} e^{\frac{i\hbar}{2m}(k_2^2 - k_1^2)t} c_{2,\vec{k} + \vec{k}_1 - \vec{k}_2}$$

$$\dot{c}_{1,\vec{k}} \approx i \frac{\Omega_1^* \Omega_2}{\Delta} e^{i\delta t} c_{2,\vec{k} + \vec{k}_1 - \vec{k}_2}$$

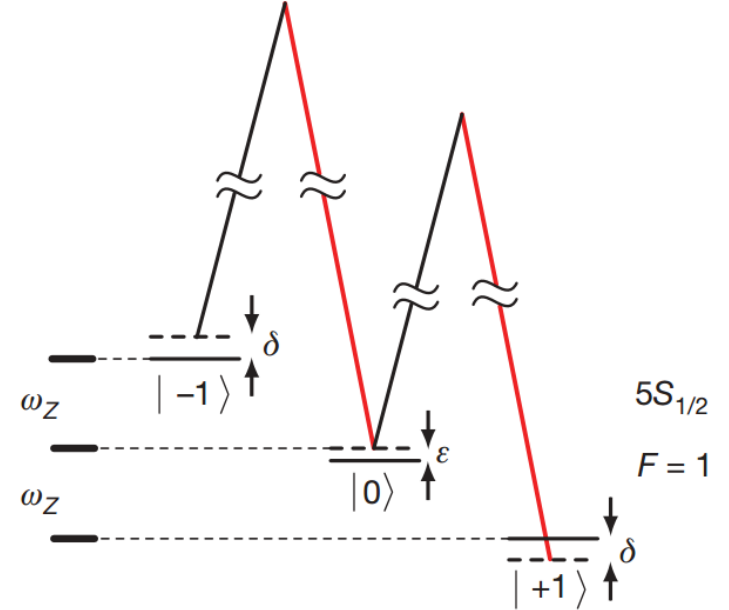
$$\dot{c}_{1,\vec{k}} \approx i \frac{\Omega_1^* \Omega_2}{\Delta} e^{i\delta t} c_{2,\vec{k}+\vec{k}_1-\vec{k}_2}$$

$$\dot{c}_{2,\vec{k}+\vec{k}_1-\vec{k}_2} \approx i \frac{\Omega_2^* \Omega_1}{\Delta} e^{-i\delta t} c_{1,\vec{k}}$$

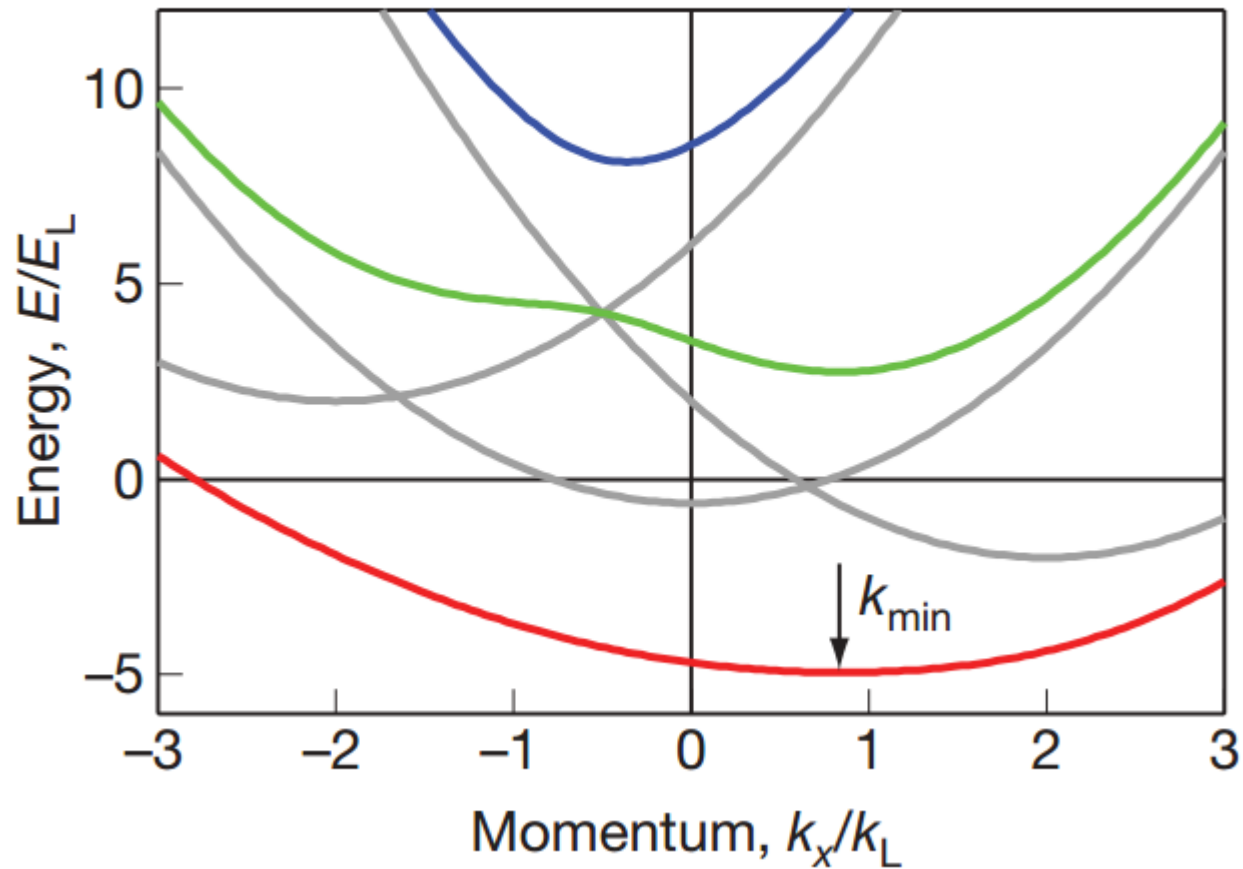
$$c_{1,\vec{k}} = e^{\frac{i\delta}{2}t} c'_{1,\vec{k}} \quad c_{2,\vec{k}} = e^{-\frac{i\delta}{2}t} c'_{2,\vec{k}}$$

$$\dot{c}'_{1,\vec{k}} \approx -\frac{i\delta}{2} c_{1,\vec{k}} + i \frac{\Omega_1^* \Omega_2}{\Delta} c'_{2,\vec{k}+\vec{k}_1-\vec{k}_2}$$

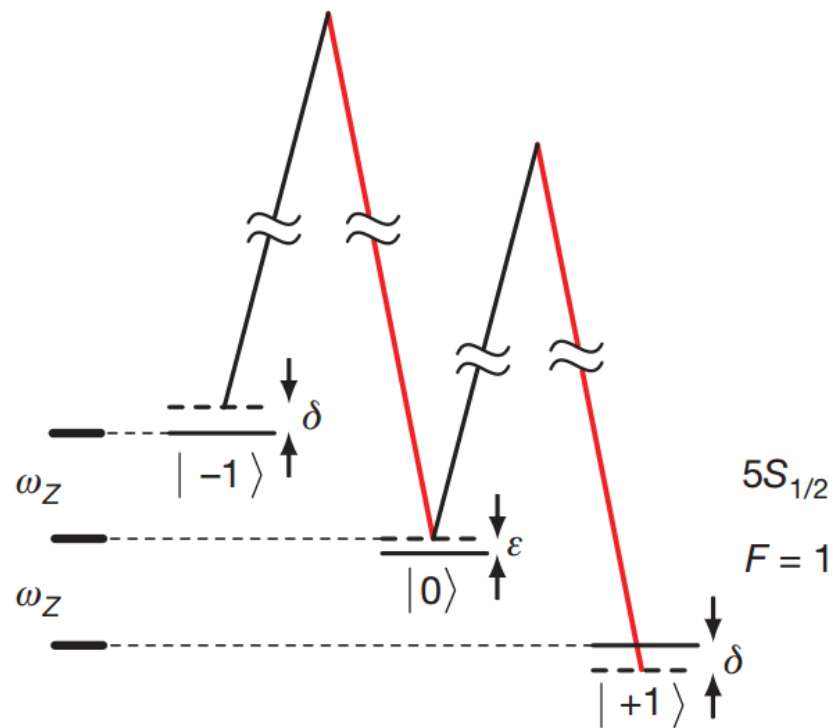
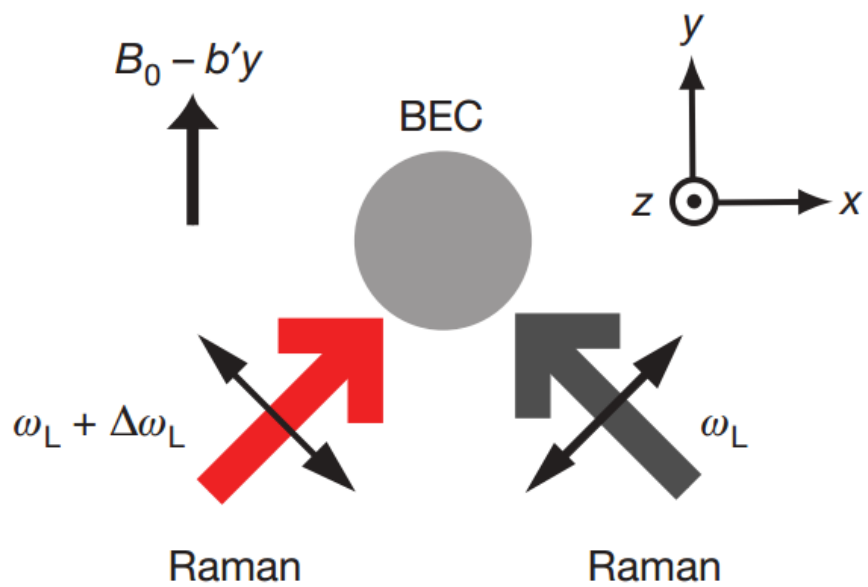
$$\dot{c}'_{2,\vec{k}+\vec{k}_1-\vec{k}_2} \approx \frac{i\delta}{2} c_{2,\vec{k}} + i \frac{\Omega_2^* \Omega_1}{\Delta} c'_{1,\vec{k}}$$



c Dispersion relation, $\hbar\delta = -2E_L$



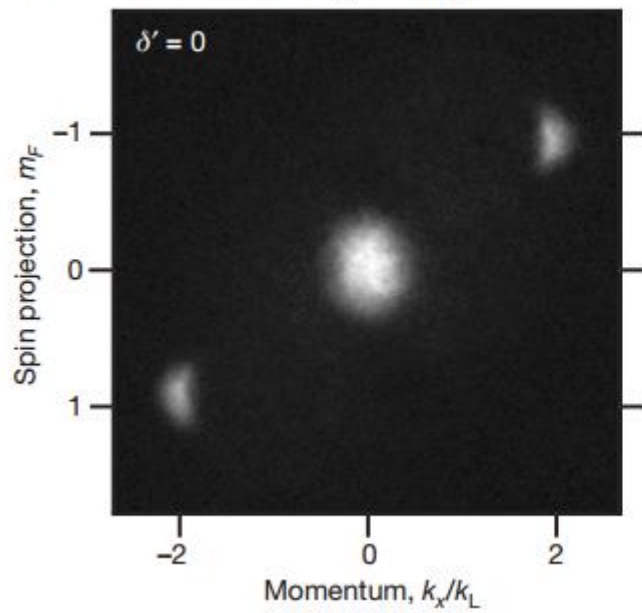
$$H = \frac{\hbar^2}{2m} (k_x - k_{\min})^2 \rightarrow \frac{\hbar^2}{2m^*} \left(k_x - \frac{q^* A^*}{\hbar} \right)^2$$



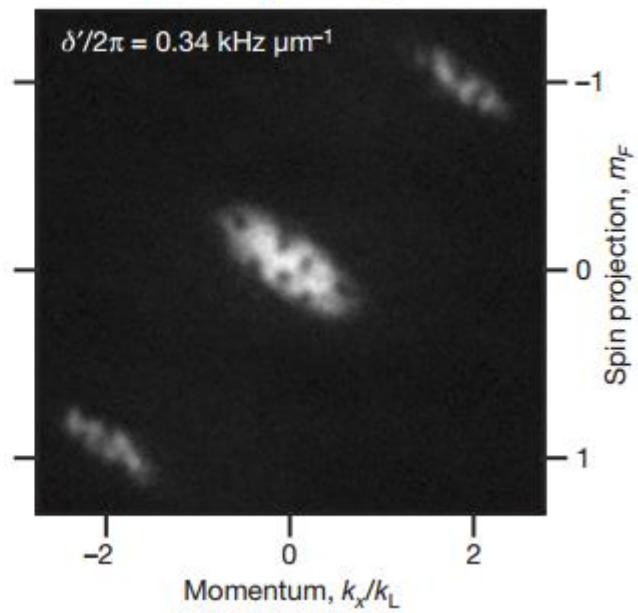
$$-B^* \hat{z} = \vec{\nabla} \times \vec{A}^*$$

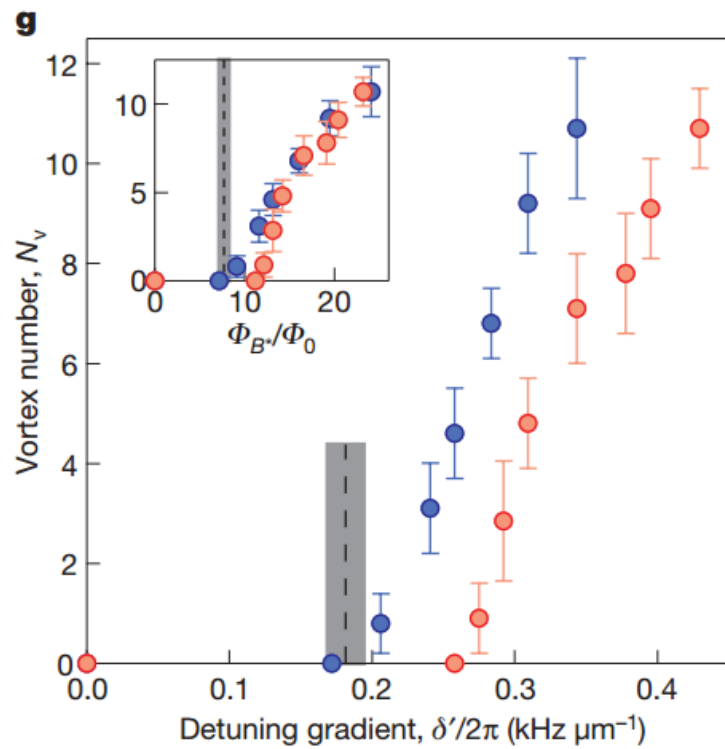
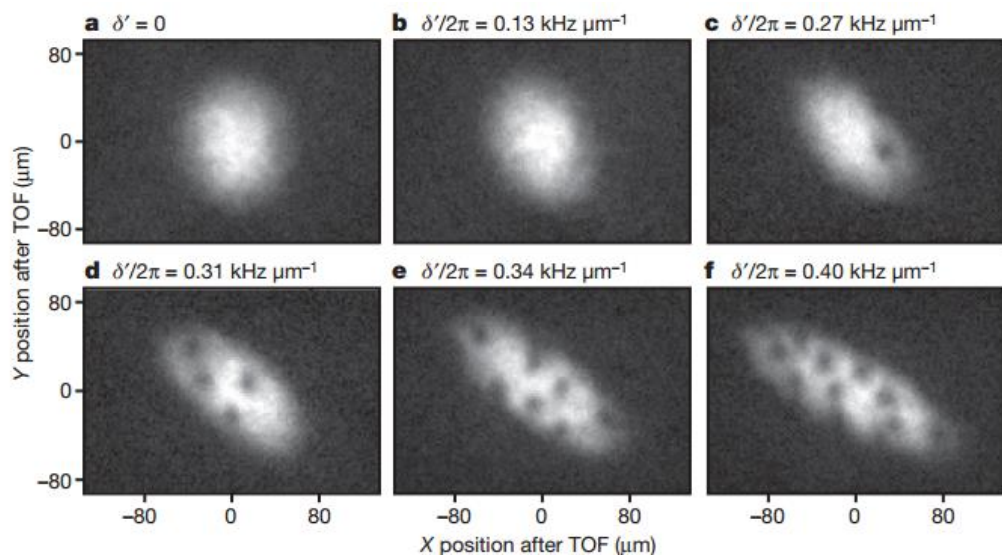
$$B_z^* = \frac{\partial A_x^*}{\partial y}$$

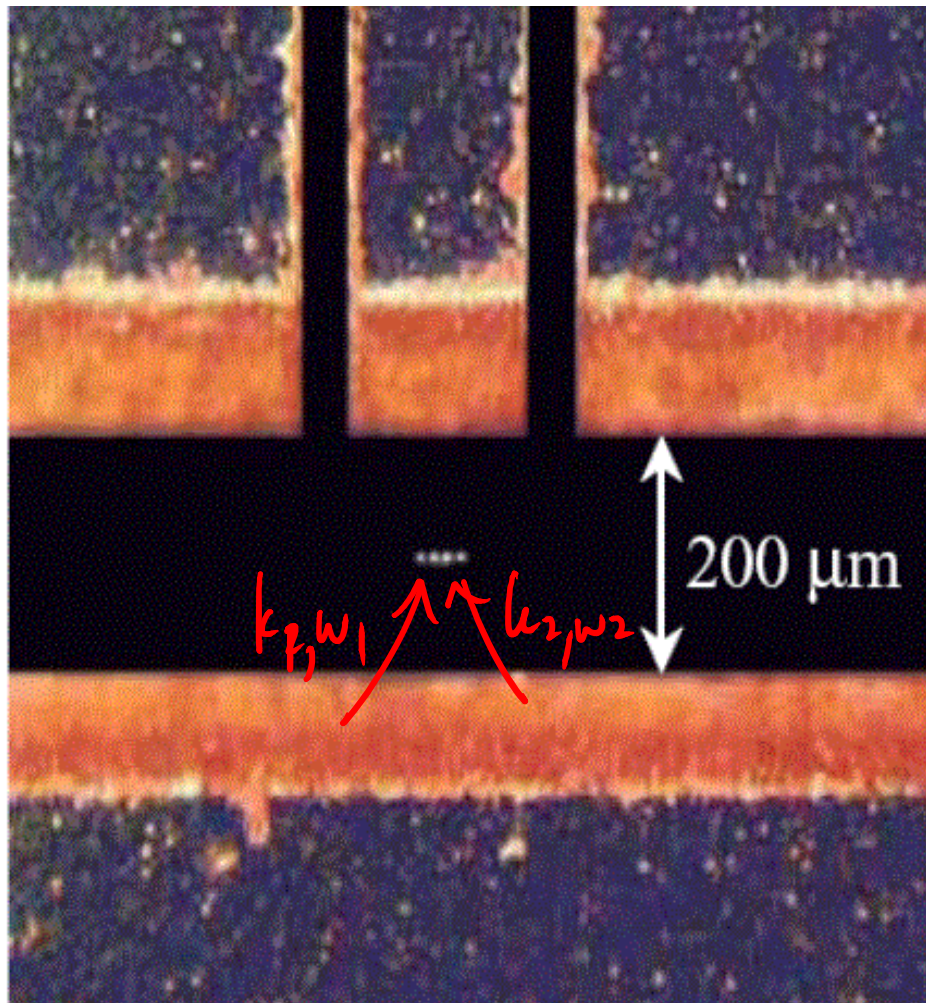
e Dressed state, $\hbar\Omega_R = 8.20E_L$



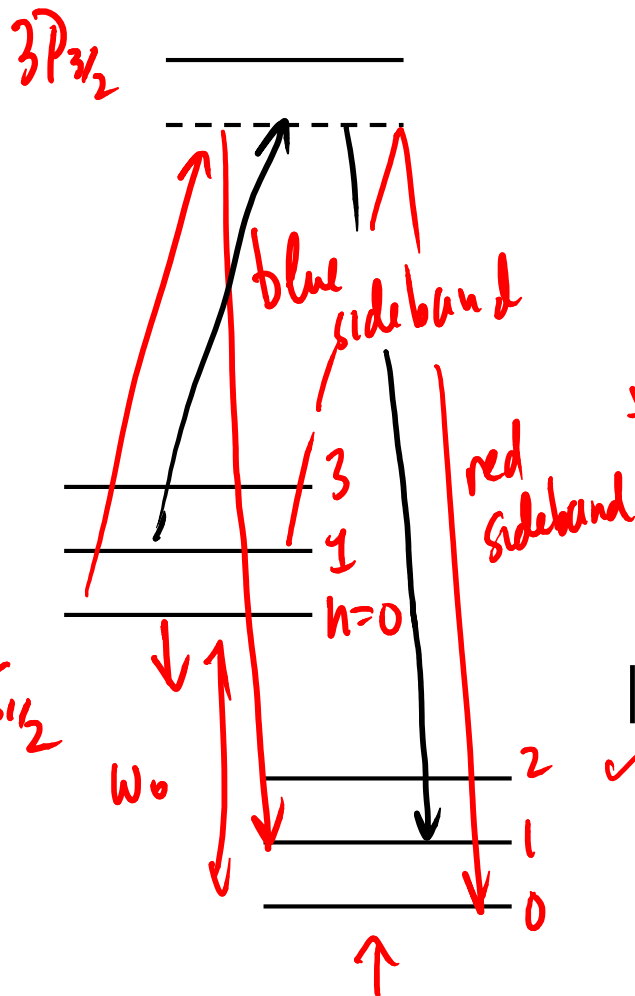
f Dressed state, $\hbar\Omega_R = 8.20E_L$







$^9\text{Be}^+$



$$H = \hbar\omega|n\rangle\langle n| + \hbar\omega_0|g_2\rangle\langle g_2| + \hbar\omega_{eg}|e\rangle\langle e|$$

$$|\psi\rangle = \sum_n (c_{1n}|g_1\rangle + c_{2n}|g_2\rangle + c_{en}|e\rangle) |n\rangle$$

$$|g_1 n\rangle \leftrightarrow |g_2 n'\rangle$$

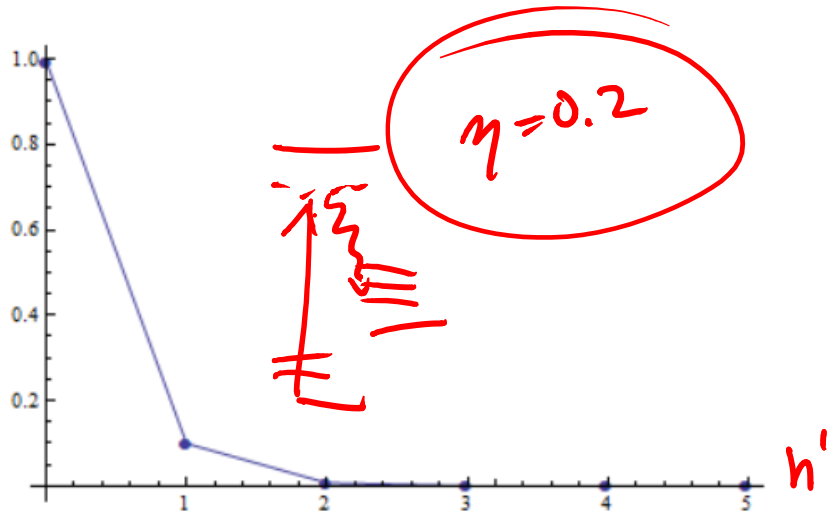
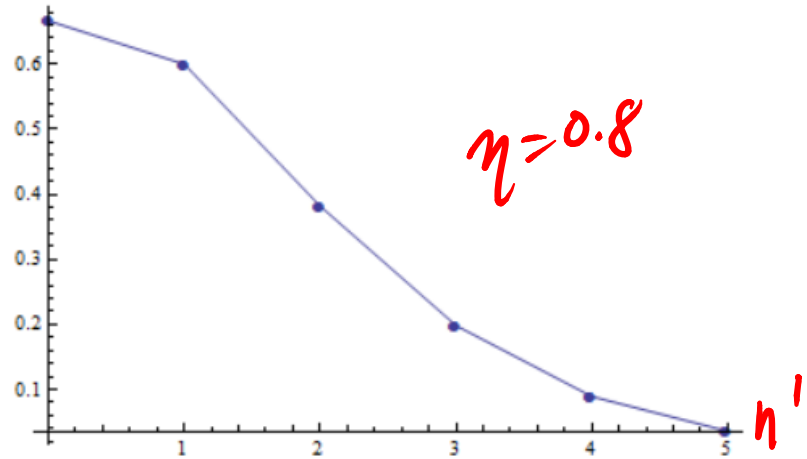
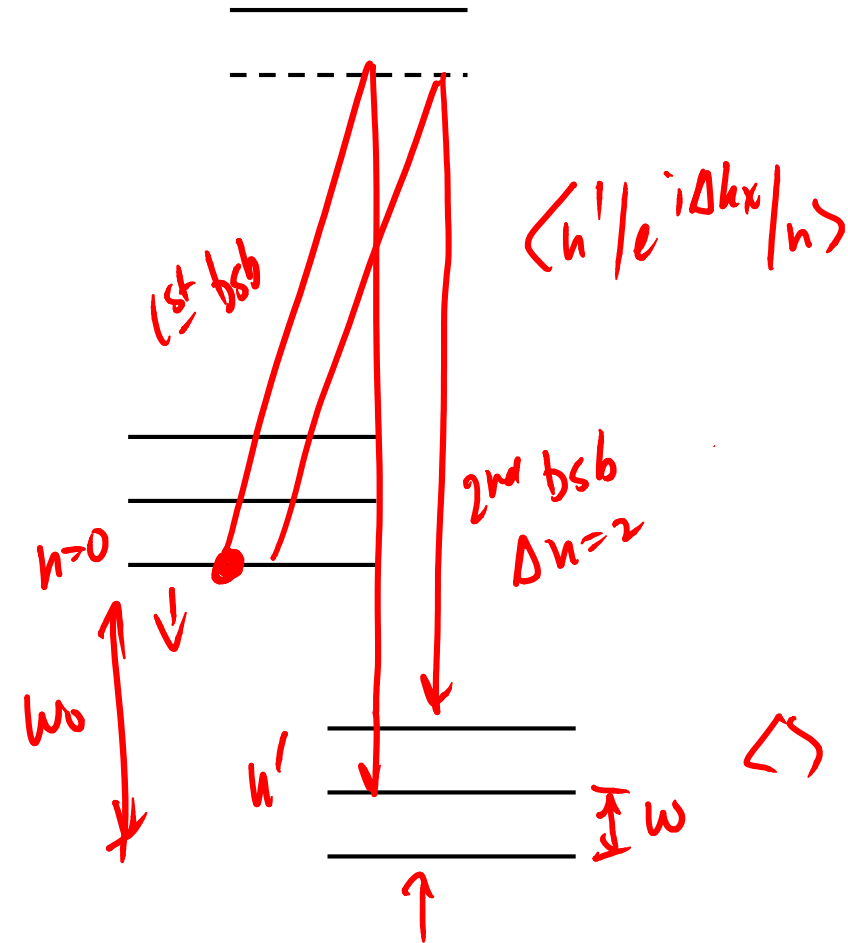
$$\Omega = \frac{\Omega_1 \Omega_2}{\Delta} \langle n' | e^{i\Delta \vec{k} \cdot \vec{x}} | n \rangle = \frac{\Omega_1 \Omega_2}{\Delta} \langle n' | e^{in(\hat{a} + \hat{a}^\dagger)} | n \rangle$$

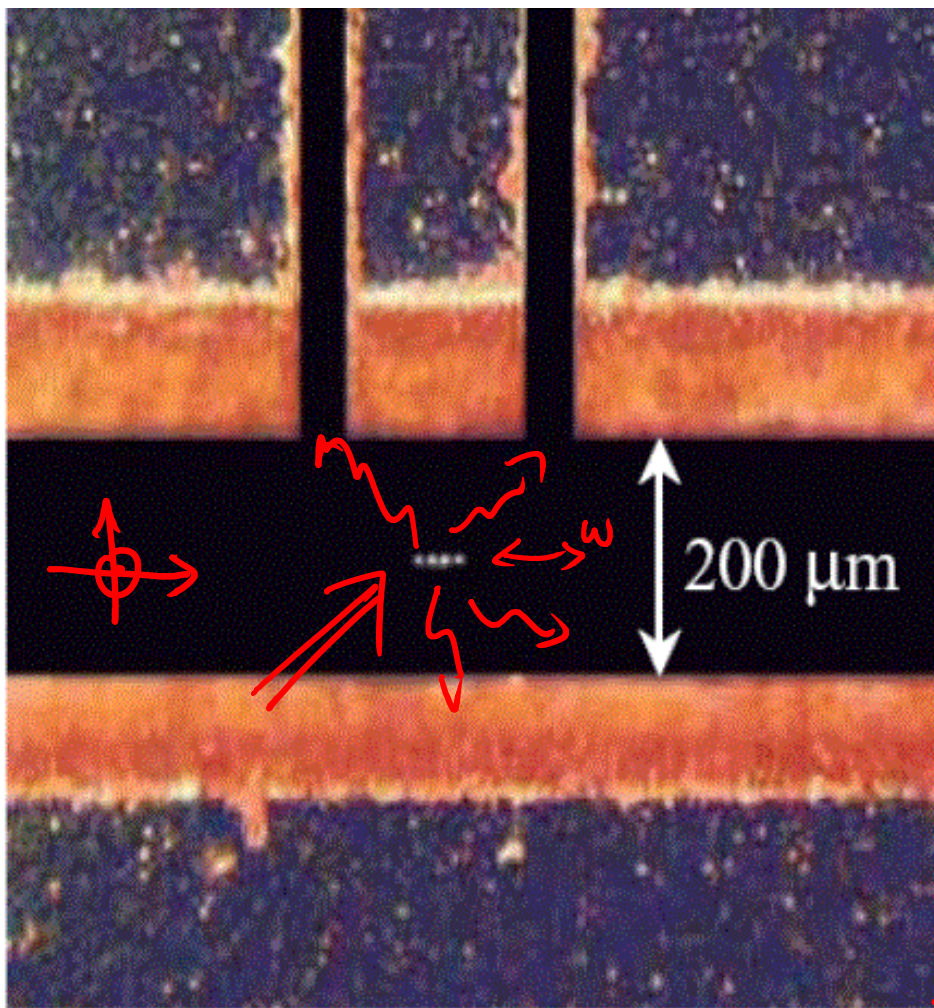
$$\eta = \Delta k_z z_0 \quad z_0 = \sqrt{\hbar/2m\omega_z}$$

Lamb-Dicke parameter

carrier: $\Delta n = 0$
 bsb: $\Delta n > 0$
 rsb: $\Delta n < 0$

$$\langle n' | e^{i\eta(\hat{a} + \hat{a}^\dagger)} | n \rangle = e^{-\frac{\eta^2}{2}} \sqrt{\frac{n_{<}!}{n_{>}!}} \eta^{|n' - n|} L_{n_{<}}^{|n' - n|} (\eta^2)$$





Doppler cooling

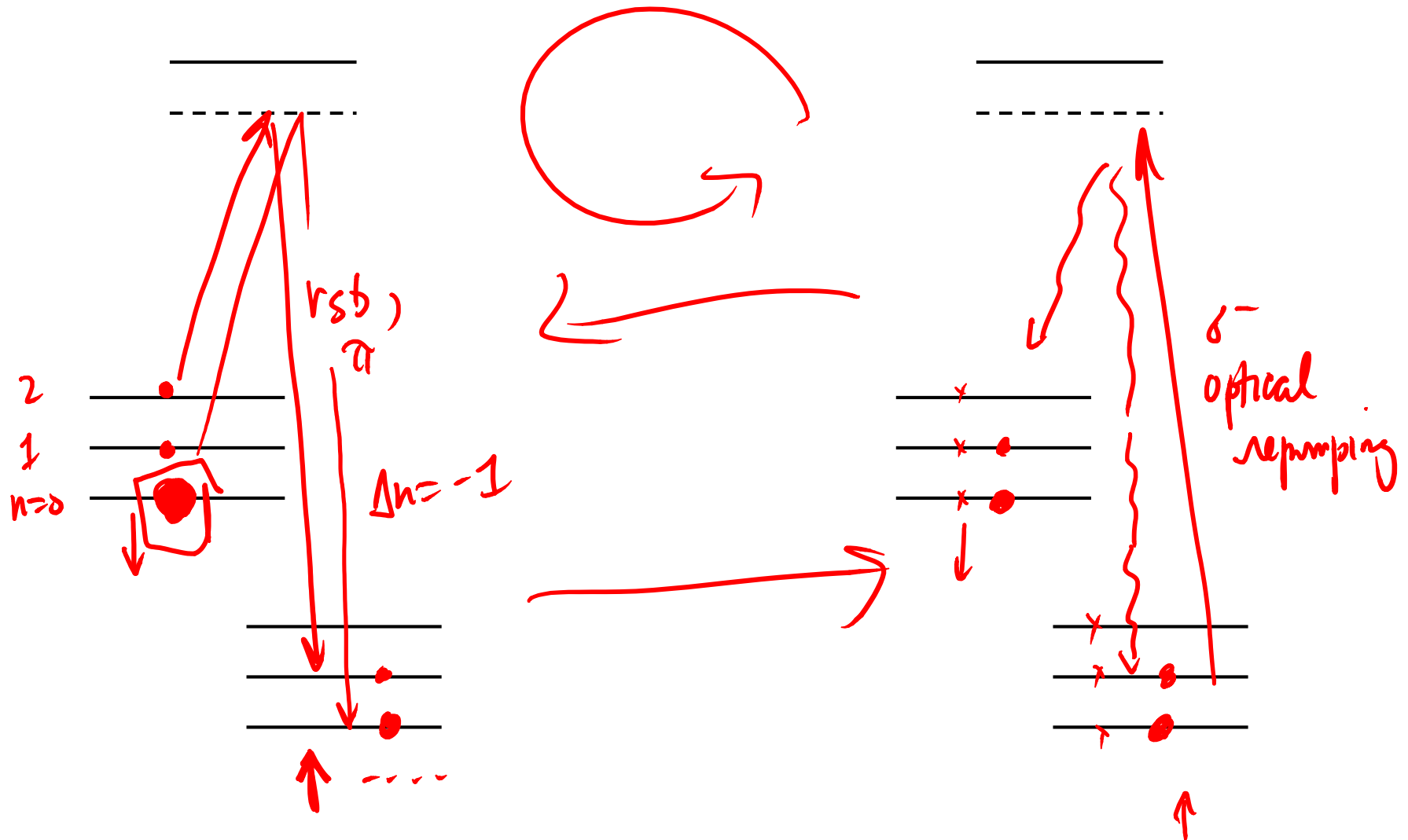
$$T_D = \frac{h\Gamma}{2k_B} \quad \approx 2\pi 20 \text{ MHz}$$

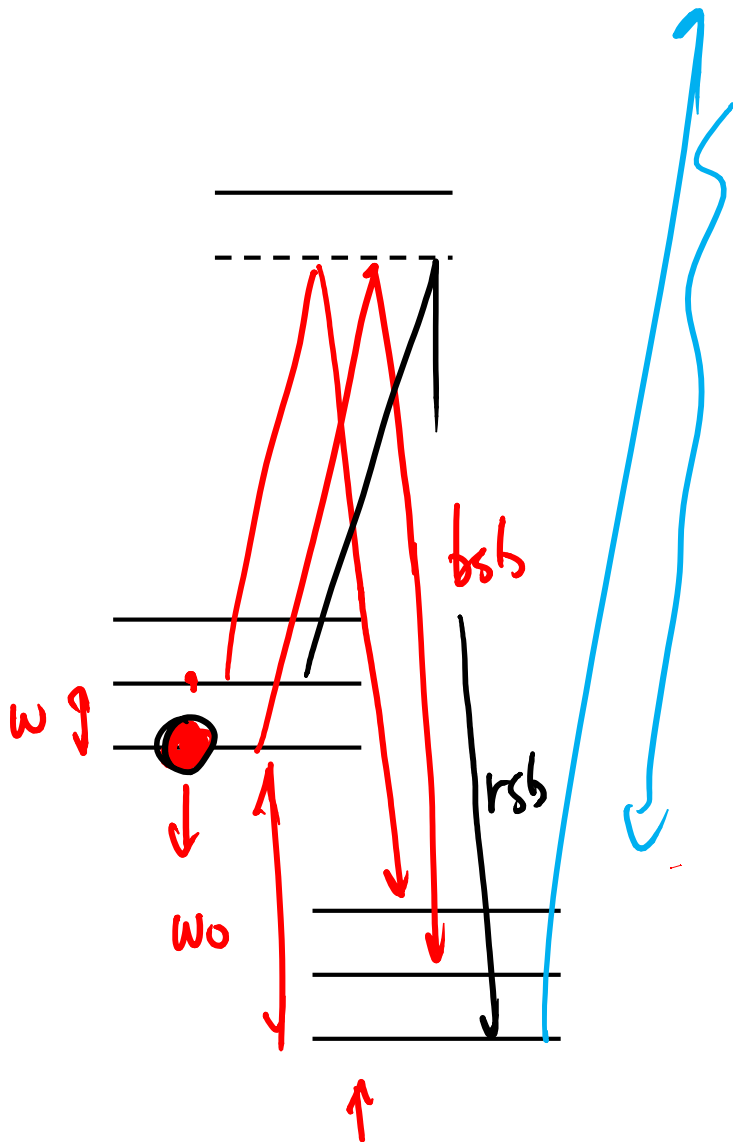
$$T_D = 3 \text{ mK}$$

$$\omega = 2\pi 10 \text{ MHz} \quad \langle n \rangle = \sum_n e^{-\frac{h\hbar\omega}{kT} \cdot n} \quad \langle n \rangle = 6$$

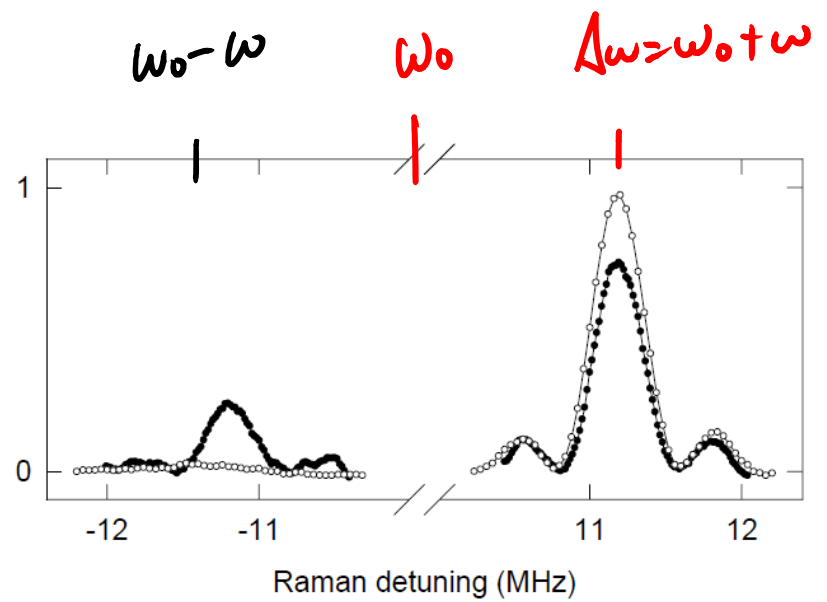
$$\sum_n e^{-\frac{h\hbar\omega}{kT} \cdot n}$$

resolved sideband cooling - Lamb-Dicke limit, $\eta \ll 1$





$P_{\uparrow}$

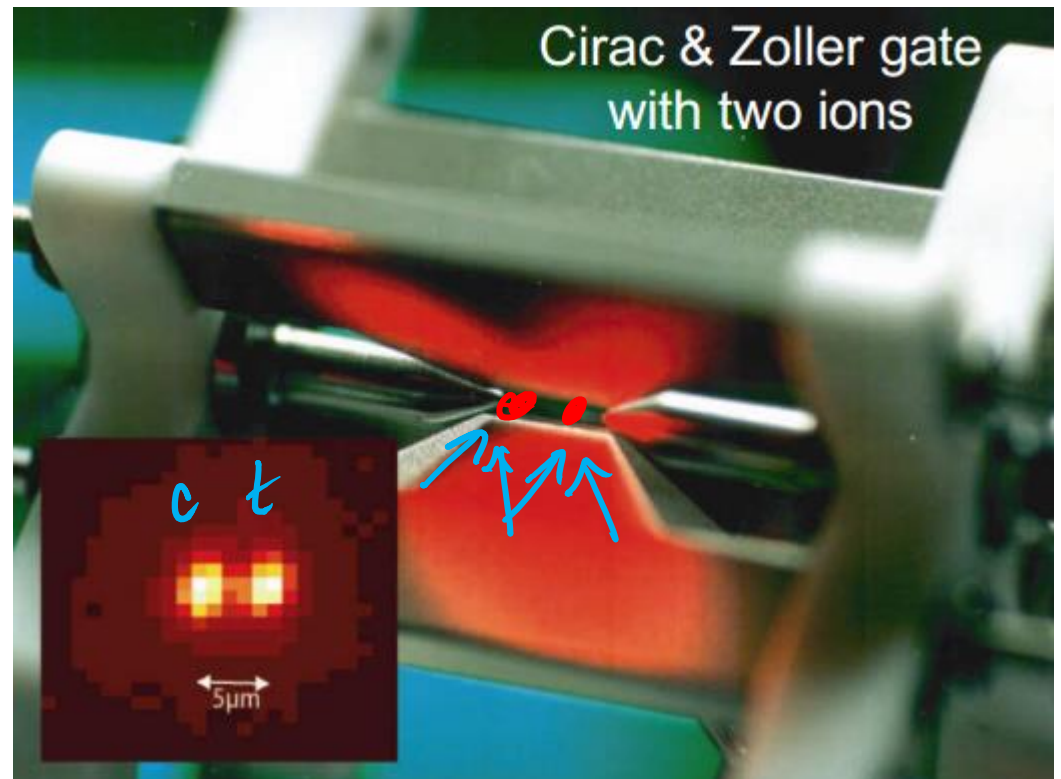


Realization of the Cirac–Zoller controlled-NOT quantum gate

Ferdinand Schmidt-Kaler, Hartmut Häffner, Mark Riebe, Stephan Gulde, Gavin P. T. Lancaster, Thomas Deuschle, Christoph Becher, Christian F. Roos, Jürgen Eschner & Rainer Blatt

Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria

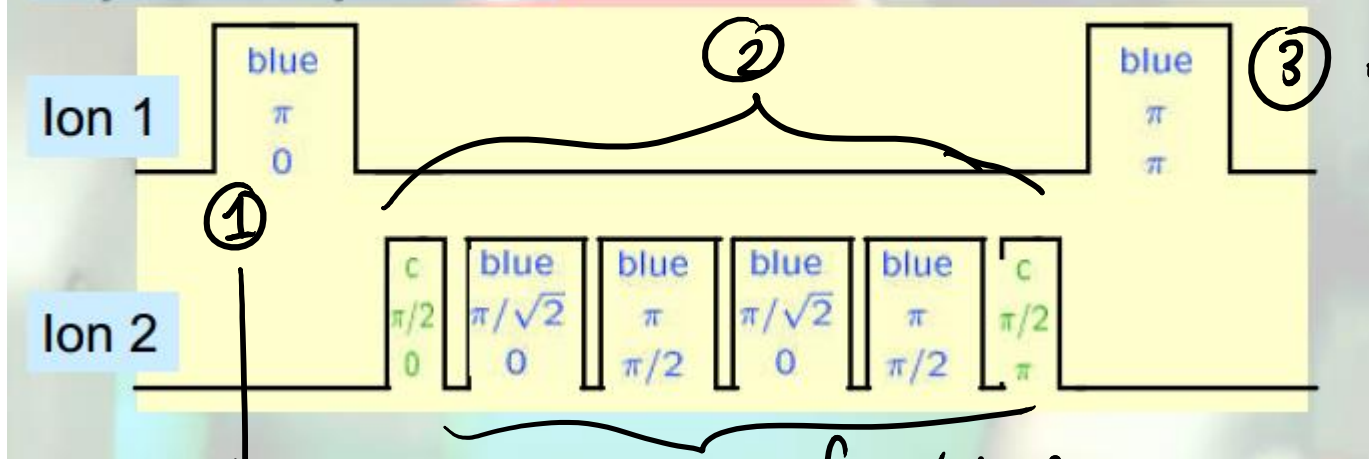
NATURE | VOL 422 | 27 MARCH 2003 | www.nature.com/nature



pulse sequence:

control

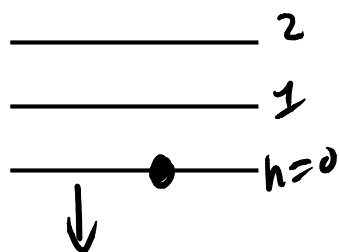
target



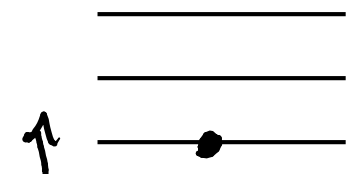
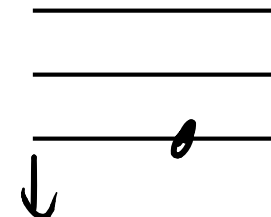
unmap motion of c

c → motion

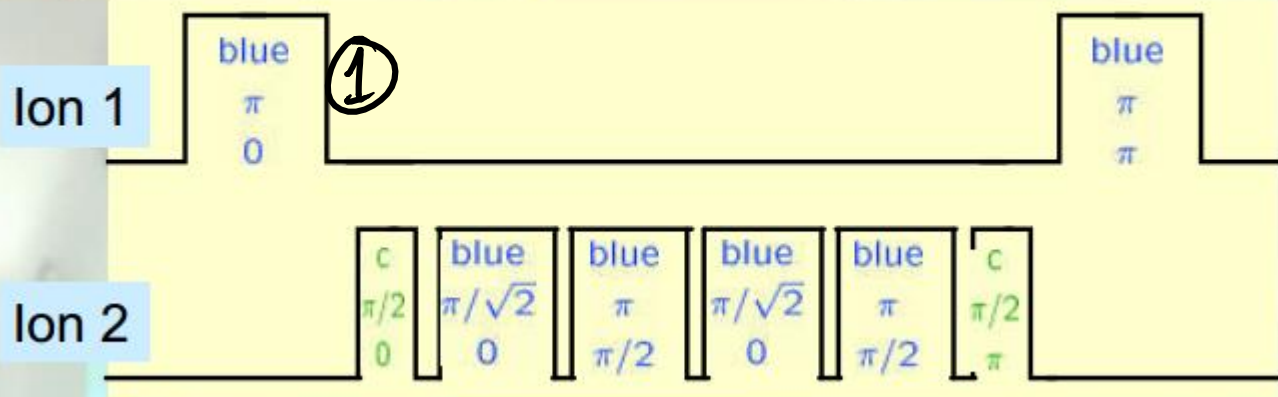
motion: flip state of t



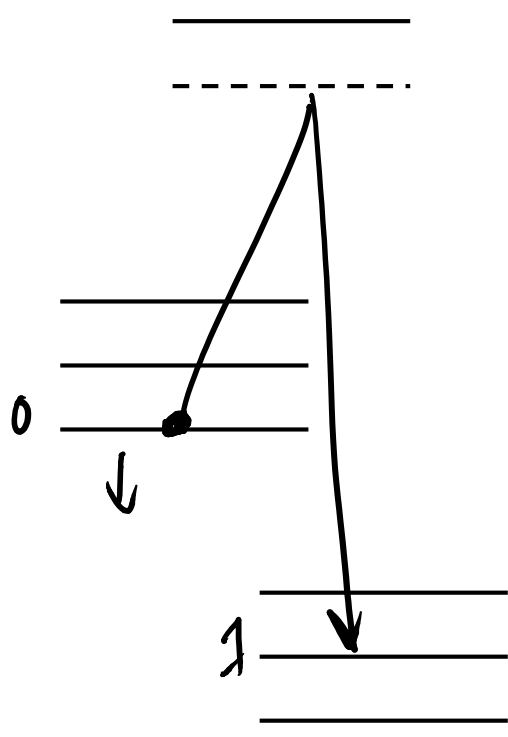
$$\begin{array}{cc}
 \begin{array}{c} c \ t \\ \hline \downarrow 0 \ \downarrow 0 \\ \downarrow 0 \ \uparrow 0 \\ \uparrow 0 \ \downarrow 0 \\ \uparrow 0 \ \uparrow 0 \end{array} & \text{CNOT} \Rightarrow \\
 \begin{array}{c} c' \ t' \\ \hline \downarrow 0 \ \downarrow 0 \\ \downarrow 0 \ \uparrow 0 \\ \uparrow 0 \ \uparrow 0 \\ \uparrow 0 \ \downarrow 0 \end{array}
 \end{array}$$



pulse sequence:

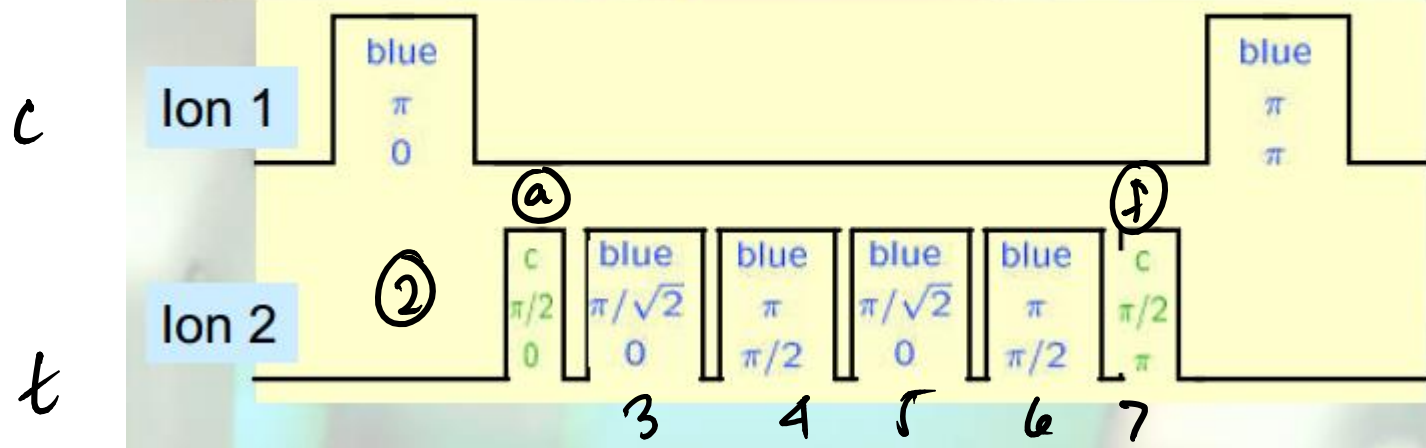


control



c	t		c'	t'
↓0	↓0	①	↑1	↓1
↓0	↑0	⇒	↑1	↑1
↑0	↓0		↑0	↓0
↑0	↑0		↑0	↑0

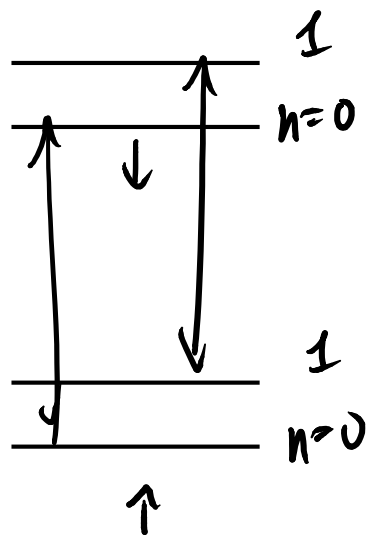
pulse sequence:



c



t

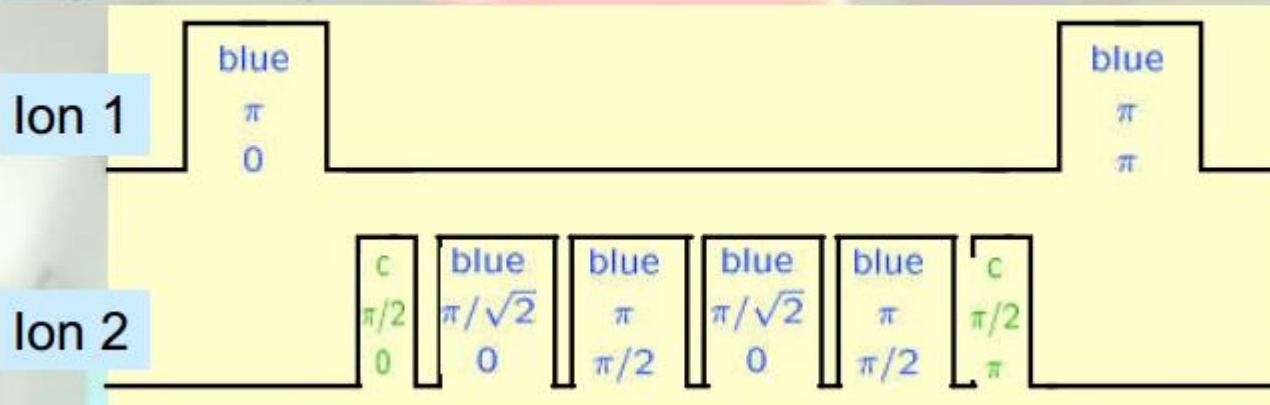


t

$$\downarrow_n \rightarrow (\downarrow + \uparrow)_n \quad (a)$$

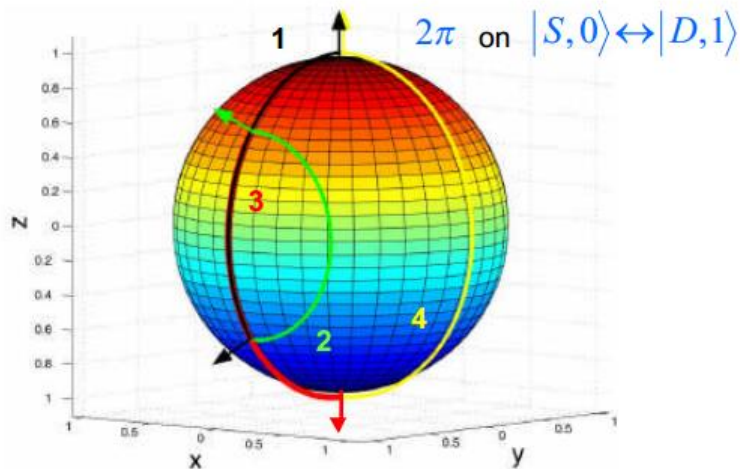
$$\uparrow_n \rightarrow (\downarrow - \uparrow)_n$$

pulse sequence:



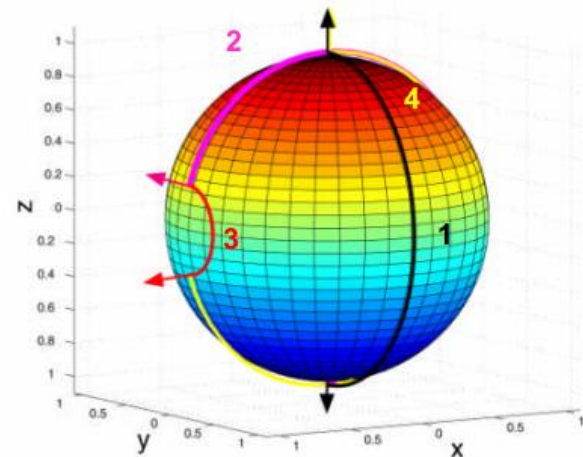
Composite phase gate (2π rotation)

$$R(\theta, \phi) = R_1^+ (\pi, \pi/2) R_1^+ (\pi/\sqrt{2}, 0) R_1^+ (\pi, \pi/2) R_1^+ (\pi/\sqrt{2}, 0)$$



Population of $|S,1\rangle - |D,2\rangle$ remains unaffected

$$R(\theta, \phi) = R_1^+ (\pi\sqrt{2}, \pi/2) R_1^+ (\pi, 0) R_1^+ (\pi\sqrt{2}, \pi/2) R_1^+ (\pi, 0)$$



pulse sequence:

Ion 1

blue

π

0

blue

π

π

Ion 2

c

$\pi/2$

0

blue

$\pi/\sqrt{2}$

0

blue

π

$\pi/2$

blue

$\pi/\sqrt{2}$

0

blue

π

$\pi/2$

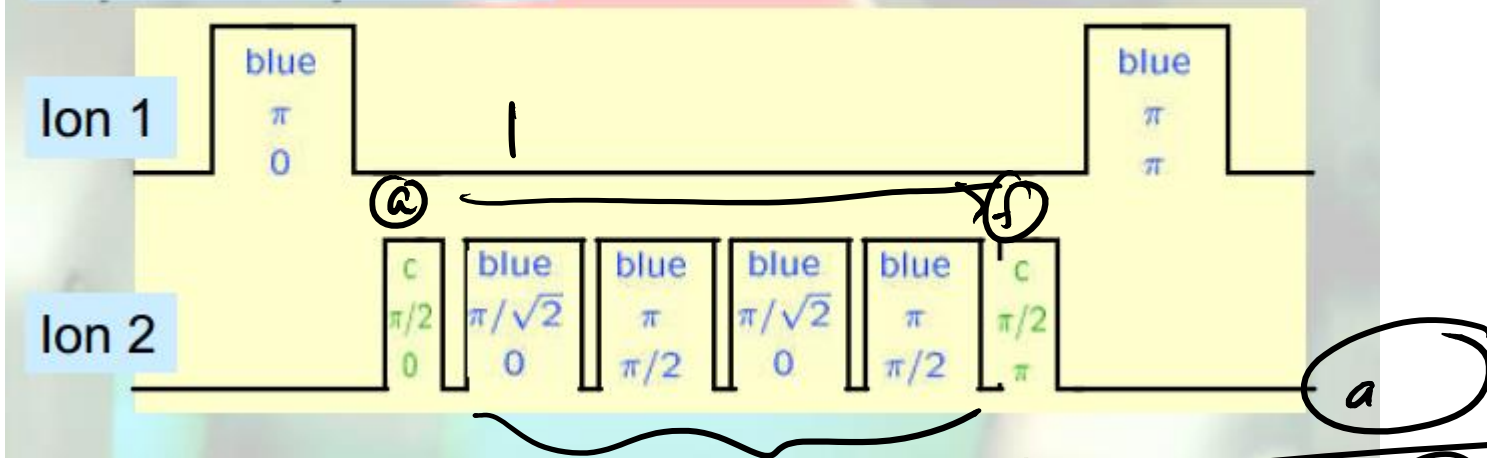
c

$\pi/2$

π



pulse sequence:



$$\begin{array}{c} \underline{t} \\ \downarrow 0 \\ \downarrow 1 \\ \uparrow 0 \\ \uparrow 1 \end{array} \Rightarrow$$

$$\begin{array}{c} \underline{t} \\ -\downarrow 0 \\ -\downarrow 1 \\ \uparrow 0 \\ -\uparrow 1 \end{array}$$

$$\begin{array}{c} \underline{t} \\ \downarrow 0 \\ \downarrow 1 \\ \uparrow 0 \\ \uparrow 1 \end{array} \xrightarrow{(a)} \begin{array}{cc} (\downarrow \uparrow)_0 & (\downarrow \uparrow)_0 \\ (\downarrow \uparrow)_1 & (-\downarrow -\uparrow)_1 \\ (\downarrow -\uparrow)_0 & (-\downarrow -\uparrow)_0 \\ (\downarrow -\uparrow)_0 & (-\downarrow \uparrow)_1 \end{array}$$

(b-e)

pulse sequence:

